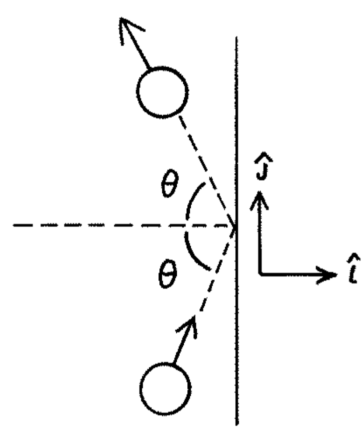
Exam 2 Sample Questions

1. You observe a wheel that is rotating clockwise (as you view it) and speeding up its rate of rotation. Which is true?

A. The angular velocity points away from you and the angular acceleration is zero.
B. The angular velocity points away from you and the angular acceleration points toward you.
C. Both the angular velocity and the angular acceleration point away from you.
D. The angular velocity points towards you and the angular acceleration is zero.
C. Both the angular velocity and the angular acceleration point towards you.

1)  If the wheel is rotating clockwise, its angular velocity vector points away from us (righthand rule). The wheel is speeding up, so its angular acceleration vector points in the same direction as $\vec{\omega}$ (also away from us).

2. A ball of mass *m* strikes a massive wall at speed v_o at an angle θ with the normal to the wall. It bounces off with the same speed and angle. If the ball is in contact with the wall for a time *t*, what is the magnitude of the impulse applied to the ball by the wall?



A. $2mv_0$

B. $2mv_o \sin\theta$

C. $2mv_o \cos\theta$

D. $mv_0 \tan\theta$

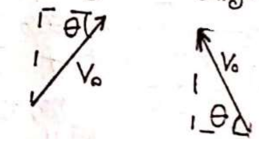
E. mv_0/t

2) $\vec{J} = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = (p_{fx} - p_{ix})\hat{\imath} + (p_{fy} - p_{iy})\hat{\jmath}$

$= (mv_0\cos\theta - mv_0\cos\theta)\hat{\imath} + (mv_0\sin\theta - mv_0\sin\theta)\hat{\jmath}$

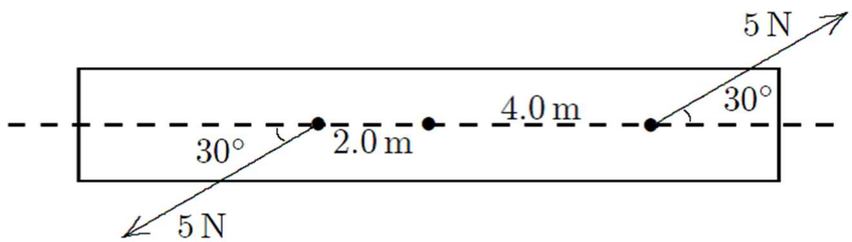
$\vec{J} = -2mv_0\cos\theta\,\hat{\imath}$

$\boxed{|\vec{J}| = 2mv_0\cos\theta}$



Note that the outgoing x component is negative.

3. A block is pivoted about its center. A 5.0 N force is applied 4.0 m from the center and another at 2.0 m from the center. Both forces act at 30˚ as shown. The magnitude of the net torque about the center is



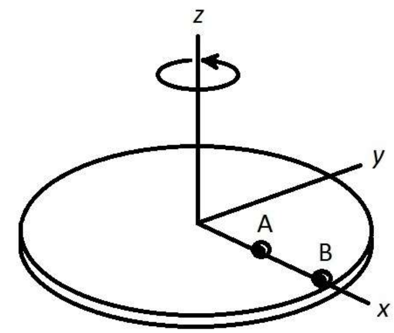
A. 0 N·m
B. 4.3 N·m
C. 5.0 N·m
D. 13 N·m
E. 15 N·m

3) Both torques point out of the page (righthand rule). The net torque is

$\vec{\tau}_{net} = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 = ((4.0\text{m})(5\text{N})\sin 30° + (2.0\text{m})(5\text{N})\sin 30°)\hat{z}$

$\boxed{\vec{\tau}_{net} = 15\,\text{N·m}\,\hat{z}}$

4. Point A is halfway between point B and the axis of rotation of a solid circular platform as shown. Given these three statements;



I. Point A has twice the **angular** velocity of point B.
II. The **angular** velocity vector is in the positive $\hat{x}$ direction.
III. Point A and point B have the same **angular** acceleration.

Which of the above statements are true?

A. I only
B. II only
C. III only
D. I and III only
E. II and III only

4) Angular kinematic quantities ($\Delta\theta$, ω, α) are the same at all points on a rigid body, while tangential quantities (arc length, v, a) are proportional to radial distance ($s = r\Delta\theta$, $v = r\omega$, $a = r\alpha$).

Statement I is false, since A and B have the same $\vec{\omega}$.
II is false, since $\vec{\omega}$ points along $+\hat{z}$ (by righthand rule).
Only III is true, since all points on the disk have the same $\vec{\alpha}$.

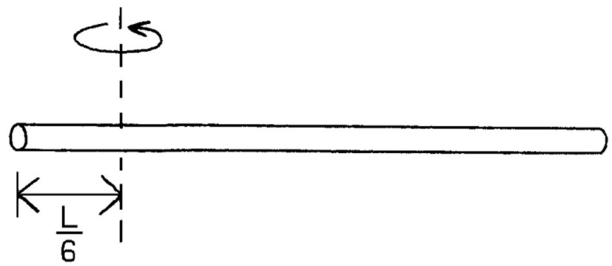
5 Consider a long thin rod of uniform density (mass M and length L) that is rotated about an axis that is perpendicular to its length and located one sixth of the length from one end, as shown in the diagram. What is the moment of inertia for the rod about this axis?

A. $\frac{7}{36}ML^2$

B. $\frac{11}{36}ML^2$

C. $\frac{13}{36}ML^2$

D. $\frac{15}{36}ML^2$



E. None of the above

1) We'll use the parallel axis theorem:

$$I = I_{com} + md^2$$

$$I = \frac{1}{12}ML^2 + M\left(\frac{2}{6}L\right)^2$$

$$\boxed{I = \frac{7}{36}ML^2}$$

When using the parallel axis theorem, always start from I about an axis through the center of mass $\left(\frac{1}{12}ML^2\right)$, not the end $\left(\frac{1}{3}ML^2\right)$.

6 A wheel is spinning at 18 rad/s but is slowing with a **time-varying** angular acceleration that has a magnitude given by (4.0 rad/s^3) t. The wheel will slow down to half its initial speed in a time of

A. 1.5 s
B. 2.1 s
C. 2.3 s
D. 3.0 s
E. 4.5 s

6 Time-varying α, so we need to integrate.

$$\omega(t) = \int \alpha(t)\,dt = \int \left(-4.0\,\frac{\text{rad}}{s^3}\right)t\,dt = \left(-2.0\,\frac{\text{rad}}{s^3}\right)t^2 + c$$

Because $\omega(0\,s) = 18$ rad/s, $\quad c = 18$ rad/s

$$\omega(t) = -\left(2.0\,\frac{\text{rad}}{s^3}\right)t^2 + 18\,\text{rad/s}$$

(The negative sign on the time-dependent term is because the wheel is slowing down).

We now solve for t such that $\omega(t) = \frac{1}{2}\omega_0$:

$$9\,\frac{\text{rad}}{s} = -\left(2.0\,\frac{\text{rad}}{s^3}\right)t^2 + 18\,\text{rad/s}$$

$$9 = 2.0\,\frac{1}{s^2}\,t^2$$

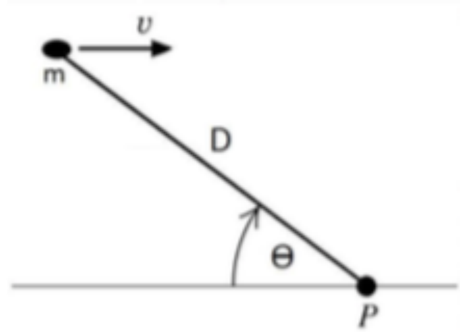
$$t = \sqrt{4.5\,s^2}$$

$$\boxed{t = 2.1\,s}$$

7

A rock is thrown into the air. At the very top of its trajectory, the velocity of the rock is level with the ground and has a magnitude v. A person standing on the ground at point P observes the rock at angle θ above the horizon as shown. The angular momentum of the rock about the point P at the moment it's at the top of its trajectory has a magnitude given by

A. $\boldsymbol{mv}$
B. $\boldsymbol{mvD}$ $\mathbf{sin}\ \boldsymbol{\theta}$
C. $\boldsymbol{mvD}$ $\mathbf{cos}\ \boldsymbol{\theta}$
D. $\boldsymbol{mvD}$ $\mathbf{tan}\ \boldsymbol{\theta}$
E. 0 (zero)



$\vec{L} = \vec{r} \times m\vec{v}$. The cross product will pick out the part of $\vec{r}$ that is $\perp$ to $\vec{v}$ – here, that's the vertical component, opposite the given angle.

$$\boxed{|\vec{L}| = mvD\sin\theta}$$

8 A uniform rod of mass m and length L swings downward, pivoting about a point at the top of the rod. Which of the following is a true statement as the rod swings freely downward?

A. The angular momentum of the rod about the pivot is conserved.

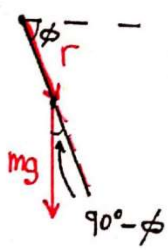
B. The rod has a constant torque acting on it as it swings

C. The rod swings down with a constant angular velocity.

D. The rod swings down with a constant angular acceleration.

E. None of the above.

6)



The angular momentum of the rod changes due to the external torque exerted by gravity. That torque depends on the angle the rod makes with the vertical, so the torque isn't constant as the rod swings down. The nonzero torque creates a nonzero $\vec{\alpha}$, so $\vec{\omega}$ isn't constant. $\vec{\alpha}$ is directly proportional to $\vec{\tau}$ ($\vec{\tau}_{net}=I\vec{\alpha}$), so non-constant torque means nonconstant $\vec{\alpha}$. None of the options are true.

9 Two identical railway cars with the same mass are travelling in the same direction along a straight and level section of rail. One car is travelling at a speed of $\boldsymbol{3v}$ and the other at a speed of $\boldsymbol{v}$. The faster car strikes the back of the slower car and they link together. What is the final speed of the combined pair?

A. $\mathbf{2.00}\,\boldsymbol{v}$
B. $\mathbf{2.24}\,\boldsymbol{v}$
C. $\mathbf{3.00}\,\boldsymbol{v}$
D. $\mathbf{3.16}\,\boldsymbol{v}$
E. $\mathbf{4.00}\,\boldsymbol{v}$

9 We use conservation of linear momentum:

$$\vec{p}_{1i} + \vec{p}_{2i} = \vec{p}_{together\,f}$$

$$m(3v) + mv = (m+m)v_f$$

$$v_f = \frac{4mv}{2m}$$

$$\boxed{v_f = 2v}$$

10 Two vectors **A** and **B** are shown. What is the direction of **A x B** ?

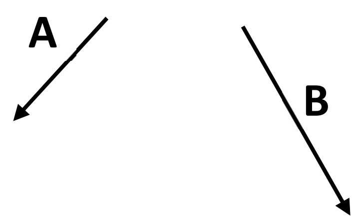
A. Into the page

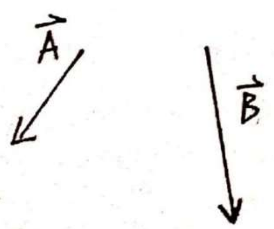
B. Out of the page

C. Along **A**

D. Along **B**

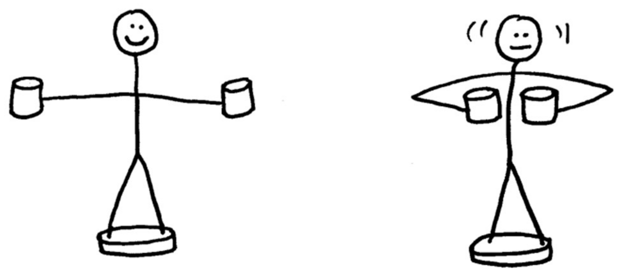
E. Towards the bottom of the page





$\vec{A} \times \vec{B}$ points out of the page (righthand rule – point your fingers along $\vec{A}$, then curl along $\vec{B}$).

11 A student stands upon a freely-rotating platform with some initial angular speed, holding weights in both hands. The student then pulls the weights inwards. Which of the following is a true statement concerning the student and the weights?



A. The moment of inertia decreases and the angular speed also decreases

B. The moment of inertia increases and the angular speed also increases

C. The moment of inertia increases, and he angular speed decreases

D. The moment of inertia decreases and the angular speed increases

E. None of the above.

Bringing some mass in closer to the axis of rotation means that the moment of inertia has decreased. Since no net external torque was involved, the angular momentum of the system is unchanged. $\vec{L} = I\vec{\omega}$. If I decreased but $\vec{L}$ was unchanged, the angular speed must have increased.

12 The vector torque about the origin due to a force $\vec{F} = (1N)\hat{i} + (-3N)\hat{j}$ acting at the point $\vec{R} = (2m)\,\hat{i}$ would be

A. $\vec{\tau} = (2N \cdot m)\,\hat{k}$

B. $\vec{\tau} = (-6N \cdot m)\hat{k}$

C. $\vec{\tau} = (2N \cdot m)\,\hat{i}$

D. $\vec{\tau} = (6N \cdot m)\hat{k}$

E. $\vec{\tau} = \vec{0}\ N{\cdot}m$

3) We'll calculate the cross product by components.

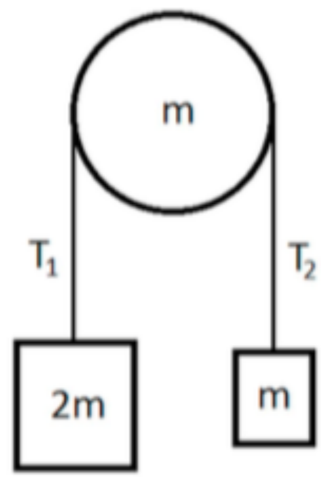
$$\vec{\tau} = \vec{r} \times \vec{F} = (r_yF_z - r_zF_y)\hat{\imath} + (r_zF_x - r_xF_z)\hat{\jmath} + (r_xF_y - r_yF_x)\hat{k}$$

$$\vec{r} = (2m)\hat{\imath} \qquad \vec{F} = (1N)\hat{\imath} + (-3N)\hat{\jmath}$$

$$\vec{\tau} = (0 \cdot -3N - 0 \cdot 0)\hat{\imath} + (0 \cdot 1N - 2m \cdot 0)\hat{\jmath} + (2m \cdot -3N - 0 \cdot 1N)\hat{k}$$

$$\boxed{\vec{\tau} = (-6Nm)\hat{k}}$$

13 Two blocks are suspended by a cord over a pulley. The masses of the blocks and the pulley are as indicated in the diagram. The blocks are free to move and the ideal cord moves over the pulley without slipping or sliding. There is no friction in the pulley axle. Which of the following is the correct ranking as the 2m mass accelerates downwards?



A. $mg < T_2 < T_1 < 2\,mg$

B. $mg = T_2 = T_1 = 2\,mg$

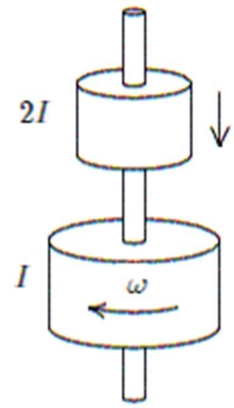
C. $mg < T_2 = T_2 < 2\,mg$

D. $mg = T_2 < T_1 = 2\,mg$

E. None of the above

If 2m is accelerating downwards, the net force on it points downwards, so its weight exceeds T_1. 2m accelerating downwards also means the angular acceleration of the pulley is counterclockwise (so $T_1 > T_2$) and m is accelerating upwards (so $T_2 > mg$). Therefore $\boxed{2mg > T_1 > T_2 > mg}$.

14 Two disks are mounted on low-friction bearings on a common shaft. The bottom disk has a rotational inertia I and is spinning with angular velocity ω. The top disk has rotational inertia 2I and is initially at rest as shown. The top disk is dropped onto the bottom disk along the shaft. They couple together and have a final common angular velocity of:



A. $\omega/3$

B. $\omega/\sqrt{3}$

C. $\omega/2$

D. $\omega/\sqrt{2}$

E. $2\omega/3$

This is a lot like critical thinking exercise 10. The total angular momentum of the system is conserved during the collision.

$$\vec{L}_{\text{bottom}_i} + \vec{L}_{\text{top}_i} = \vec{L}_{\text{together}_f}$$

$$I\omega + 2I \cdot 0 = (I + 2I)\omega_f$$

$$\omega_f = \frac{I\omega}{3I}$$

$$\boxed{\omega_f = \frac{\omega}{3}}$$

15 A force is given by $F(t) = 3.0\ t^2$. It acts on an object from t = 0 until t = 3.0 s. The object has a mass of 9.0 kg and starts from rest. What is the speed of the object at 3.0 seconds?

A. 2.4 m/s
B. 3.0 m/s
C. 5.2 m/s
D. 18 m/s
E. 27 m/s

Impulse-momentum time. Since the force is non-constant, we need to integrate.

$$\vec{J} = \Delta\vec{p}$$

$$\int_{t_i}^{t_f} \vec{F}(t)\,dt = m\vec{v}_f - m\vec{v}_i$$

$$\vec{v}_f = \vec{v}_i + \frac{1}{m}\int_{t_i}^{t_f} \vec{F}(t)\,dt$$

$$v_f = 0 + \frac{1}{9.0\,kg}\int_0^{3s} \left(3.0\,\frac{N}{s^2}\right)t^2\,dt = \frac{1}{9.0\,kg}\left(1.0\,\frac{N}{s^2}\right)t^3\Big|_{t=0}^{3s}$$

$$\boxed{v_f = 3.0\,s}$$

16 . A wheel is initially rotating at 18.0 rad/s and slows down with a **non-constant** angular acceleration that has a magnitude given by (2.00 rad/s^4) t^2. The wheel will stop in a time of

A. 2.00 s
B. 2.08 s
C. 2.62 s
D. 3.00 s
E. 9.00 s

Non-constant $\alpha \Rightarrow$ need to integrate.

$$\omega(t) = \int \alpha(t)\,dt = \int \left(-2.00 \tfrac{\text{rad}}{s^4}\right) t^2\,dt = \underbrace{-\frac{2}{3} \tfrac{\text{rad}}{s^4}}_{\text{negative because wheel is slowing down}} t^3 + \underbrace{18.0\ \text{rad/s}}_{\omega(0s) = 18.0\ \text{rad/s}}$$

negative because wheel is slowing down

$\omega(0s) = 18.0$ rad/s. We achieve this initial condition by setting our constant of integration.

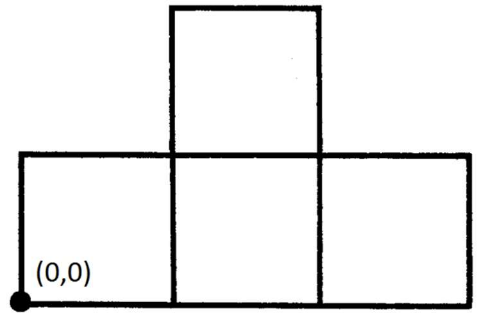
Now we solve for t such that $\omega(t) = 0$.

$$0 = -\frac{2}{3} \tfrac{\text{rad}}{s^4} t^3 + 18.0\ \text{rad/s}$$

$$t^3 = 27.0\ s^3$$

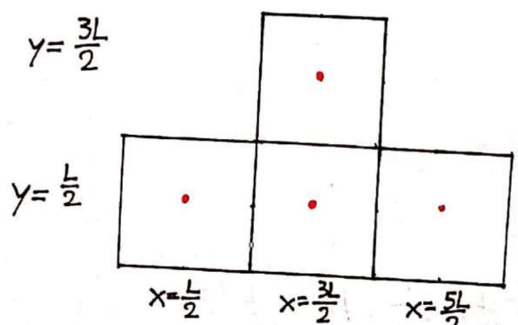
$$\boxed{t = 3.00\,s}$$

17. Four squares of equal mass and sides of length L are arranged as shown with the origin at the lower left. The (x,y) coordinates of the center of mass of the object are:



A. (3L/2, 3L/16)
B. (3L/2, 3L/8)
C. (3L/2, 3L/4)
D. (3L/2, L)
E. (3L/2, 3L/2)

We find the center of mass of the squares by finding the center of mass of their centers of mass. Since all squares have equal masses, we'll call their masses each m.



$$X_{com}=\frac{\frac{L}{2}m+\frac{3L}{2}m+\frac{3L}{2}m+\frac{5L}{2}m}{m+m+m+m}$$

$$Y_{com}=\frac{\frac{L}{2}m+\frac{L}{2}m+\frac{3L}{2}m+\frac{L}{2}m}{m+m+m+m}$$

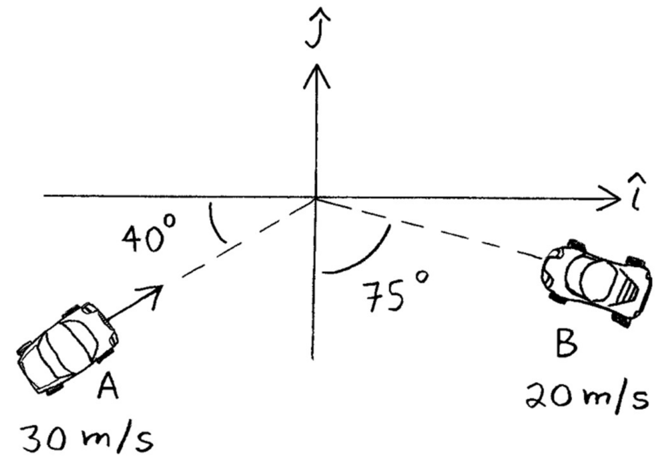
$$X_{com}=\frac{\frac{12L}{2}m}{4m}$$

$$Y_{com}=\frac{\frac{6L}{2}m}{4m}$$

$$\boxed{\vec{X}_{com}=\left(\frac{3L}{2},\ \frac{3L}{4}\right)}$$

1:

Car A of mass 800 kg and car B of mass 1200 kg are headed towards each other as shown (top view). Immediately before impact, car A has a speed of 30.0 m/s and makes and angle of 40º with the <u>x-axis</u> as shown. Car B has a speed of 20.0 m/s and makes and angle of 75º with the <u>y-axis</u> as shown. After the collision, the cars stick together and move as one.



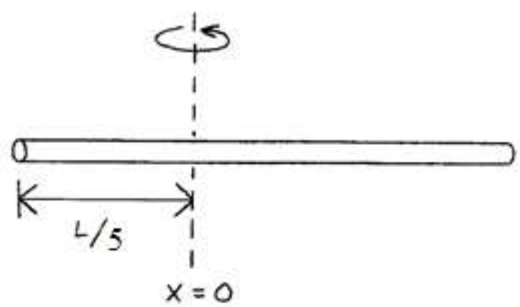
a) What is the velocity of the cars immediately after the collision, expressed in unit vector component (Cartesian) notation.

b) What is the force that Car A exerts on Car B during the collision, if the collision lasts 0.420 seconds? Express your answer in unit vector component (Cartesian) notation.

c) Is the collision elastic or inelastic? Justify you answer with some sort of calculation.

2:

A thin rod has a <u>non-uniform</u> density given by $\boldsymbol{\lambda = (4.00\frac{kg}{m^2})x + (3.00\frac{kg}{m})}$ and a total length L = 5.00 m. It is rotated about an axis located at L/5 as shown, where x = 0 is the axis of rotation.

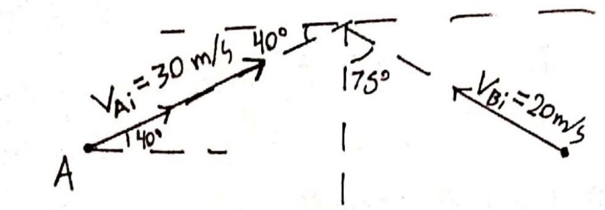


(a) Calculate the moment of inertia, clearly showing all steps to receive full credit.

(b) Calculate the total mass of the rod.

(c) Is the MOI of this rod larger or smaller than that of a uniform rod pivoted about the same axis? Justify your answer.

Long Answer 1



Translational momentum is conserved by components during collisions. The cars stick together, so we can treat the final state as one body with mass $= m_A + m_B$.

a) $$\vec{p}_{Ai} + \vec{p}_{Bi} = \vec{p}_{\substack{cars\\final}} \Rightarrow p_{Axi} + p_{Bxi} = p_{\substack{cars\,x\\final}} \text{ and } p_{Ayi} + p_{Byi} = p_{\substack{cars\,y\\final}}$$

now recall that $\vec{p} = m\vec{v}$

$$x: \; m_A V_{Axi} + m_B V_{Bxi} = (m_A + m_B) V_{\substack{cars\,x\\final}}$$

$$m_A V_{Ai} \cos 40° + m_B V_{Bi}(-\sin 75°) = (m_A + m_B) V_{\substack{cars\,x\\final}}$$

$$V_{\substack{cars\,x\\final}} = \frac{m_A V_{Ai} \cos 40° - m_B V_{Bi} \sin 75°}{m_A + m_B}$$

$$y: \; m_A V_{Ayi} + m_B V_{Byi} = (m_A + m_B) V_{\substack{cars\,y\\final}}$$

$$m_A V_{Ai} \sin 40° + m_B V_{Bi} \cos 75° = (m_A + m_B) V_{\substack{cars\,y\\final}}$$

$$V_{\substack{cars\,y\\final}} = \frac{m_A V_{Ai} \sin 40° + m_B V_{Bi} \cos 75°}{m_A + m_B}$$

$$\boxed{\vec{V}_{\substack{cars\\final}} = (-2.40\,\hat{i} + 10.8\,\hat{j}) \text{ m/s}}$$

b) We can find the average force exerted on B by using the impulse-momentum relationship:

$$\vec{F}_{\substack{av\,on\\B}} \Delta t = \Delta \vec{p}_B$$

$$\vec{F}_{\substack{av\,on\\B}} \Delta t = \vec{p}_{Bf} - \vec{p}_{Bi}$$

$$\vec{F}_{\substack{av\,on\\B}} \Delta t = m_B \vec{V}_{\substack{cars\\final}} - m_B \vec{V}_{Bi}$$

$$\vec{F}_{\substack{av\,on\\B}} = \frac{m_B}{\Delta t}\left(\vec{V}_{\substack{cars\\final}} - \vec{V}_{Bi}\right) = \frac{m_B}{\Delta t}\left[(-2.40\,\hat{i} + 10.8\,\hat{j}) \text{ m/s} - (-V_{Bi} \sin 75°\,\hat{i} + V_{Bi} \cos 75°\,\hat{j})\right]$$

$$\boxed{\vec{F}_{\substack{av\,on\\B}} = (48.4\,\hat{i} + 16.1\,\hat{j}) \text{ kN}}$$

Long Answer 1

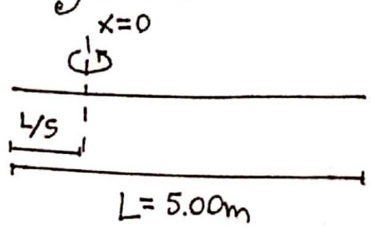
c) Because the collision is perfectly inelastic (i.e. the cars stick together), we know mechanical energy was lost. We're asked for a calculation, however, so we compute the initial and final kinetic energies

$$K_i = \frac{1}{2}m_A V_{Ai}^2 + \frac{1}{2}m_B V_{Bi}^2 = 600\,\text{kJ}$$

$$K_f = \frac{1}{2}(m_A+m_B)V_{\substack{cars\\final}}^2 = \frac{1}{2}(m_A+m_B)\left(V_{\substack{cars\,x\\final}}^2 + V_{\substack{cars\,y\\final}}^2\right) = 122\,\text{kJ}$$

$\boxed{K_f < K_i\text{, so the collision was inelastic.}}$

Long Answer 2



$$\lambda = \left(4.00\,\tfrac{kg}{m^2}\right)x + \left(3.00\,\tfrac{kg}{m}\right)$$

a) The density is nonuniform, so we have to integrate.

$$I = \int_{rod} r^2\,dm$$

$x=0$ at the axis, so $r^2 = x^2$. We express dm as $\lambda\,dx$, the integrate from end to end.

$$I = \int_{rod} r^2\,dm = \int_{-L/5}^{4L/5} x^2\left[\left(4.00\,\tfrac{kg}{m^2}\right)x + \left(3.00\,\tfrac{kg}{m}\right)\right]dx$$

$$I = \int_{-1m}^{4m} \left[\left(4.00\,\tfrac{kg}{m^2}\right)x^3 + \left(3.00\,\tfrac{kg}{m}\right)x^2\right]dx$$

$$I = \left[\left(1.00\,\tfrac{kg}{m^2}\right)x^4 + \left(1.00\,\tfrac{kg}{m}\right)x^3\right]\Big|_{x=-1m}^{4m}$$

$$\boxed{I = 320\ kg\cdot m^2}$$

b) We calculate the total mass by integrating up dm.

$$M = \int_{rod} dm = \int_{-L/5}^{4L/5} \lambda\,dx = \int_{-1m}^{4m} \left[\left(4.00\,\tfrac{kg}{m^2}\right)x + \left(3.00\,\tfrac{kg}{m}\right)\right]dx$$

$$M = \left[\left(2.00\,\tfrac{kg}{m^2}\right)x^2 + \left(3.00\,\tfrac{kg}{m}\right)x\right]\Big|_{x=-1m}^{4m}$$

$$\boxed{M = 45\ kg}$$

Long Answer 2

c) I'd expect the MoI of this rod to win out, since most of its mass is gathered far from the axis. That said, we can compare numerically w/ the uniform case, just to be safe.

The fastest way to calculate I is using the parallel axis theorem we know that $I = \frac{1}{12}ML^2$ for a rod with its axis through its midpoint, and our new axis is at a distance $d = 1.5L$ away from the midpoint.

$$I = \frac{1}{12}ML^2 + Md^2 = \frac{1}{12}(45\text{ kg})(5\text{ m})^2 + (45\text{ kg})(1.5\text{ m})^2$$

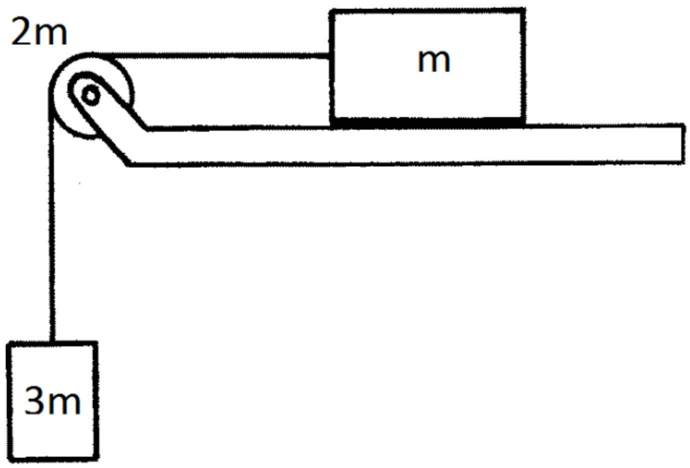
$$\boxed{I = 195\text{ kg}\cdot\text{m}^2}$$

Smaller than the $320\text{ kg}\cdot\text{m}^2$ for our nonuniform rod.

3:

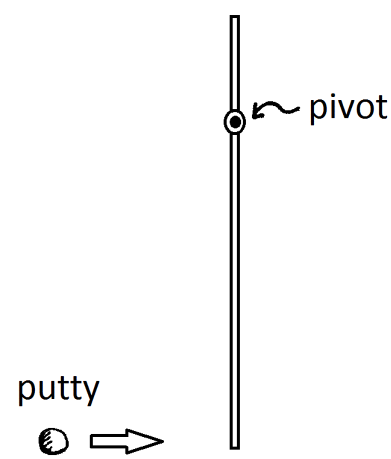
A modified Atwood's machine consists of a block of mass $\boldsymbol{m}$ sliding on a rough table with kinetic friction coefficient 0.5. A hanging mass $\boldsymbol{3m}$ pulls it by a massless string. The string passes over a pulley of mass $\boldsymbol{2m}$. The pulley is a uniform disk of radius $\boldsymbol{R}$. There is no slipping or sliding of the string over the pulley.

It is required to draw all relevant free body diagrams for the two masses and the pulley.



Find the magnitude of the acceleration of the system when released from rest, as a multiple of the gravitational acceleration g. Also find the tension in the vertical portion of the string and the tension in the horizontal portion of the string, both as multiples of mg.

4:



A long thin rod of total length 4.00 m is pivoted 1.00 m from the top end. It is hung vertically and initially at rest as shown. The rod has a non-linear mass density given by $\boldsymbol{\lambda = (3 + 15x^2)}$ kg/m, where $\boldsymbol{x = 0}$ is at the pivot. A small piece of putty of mass 0.200 kg is shot horizontally into the lower end of the rod, striking it with a speed of 20.0 m/s. The putty sticks to the rod.

a) Calculate is the moment of inertia I of the stick about the pivot.

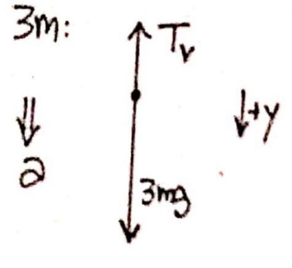
b) Calculate the angular velocity ω of the stick the instant after the putty strikes and sticks to it.

c) If you were asked to find the maximum angle that the rod/putty combination would make with the vertical after the collision, explain <u>in words</u> how you would solve this problem. Do not actually solve. Maximum 3 sentences.

Long Answer 3

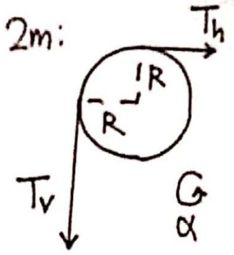
We have three unknowns (a, T_{vert}, T_{hor}). We generate a system of three equations by separately applying N2L to each of our three bodies.

3m:



$$\vec{F}_{net} = 3m\vec{a}$$
$$3mg - T_v = 3ma$$

2m:

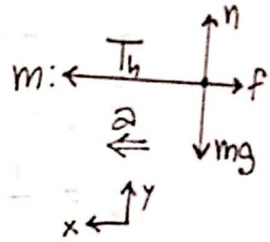


$$\vec{\tau}_{net} = I\vec{\alpha}$$
$$RT_v \sin 90° - RT_h \sin 90° = \tfrac{1}{2}MR^2\alpha$$

Because the rope doesn't slide/slip, the magnitude of its acceleration (and that of the blocks) is equal to the magnitude of the tangential acceleration of a point on the edge of the pulley $\Rightarrow \alpha = \frac{a}{R}$.

$$\cancel{R}T_v - \cancel{R}T_h = \tfrac{1}{2}M\cancel{R}a$$
$$T_v - T_h = \tfrac{1}{2}(2m)a$$
(Mass of the pulley is 2m)
$$T_v - T_h = ma$$

m:



Because the mass stays on the table, $a_y = 0 \Rightarrow F_{net_y} = 0$.

$$F_{net_y} = 0$$
$$n - mg = 0$$
$$n = mg$$

We use this to find the friction force in our x equation

$$\vec{F}_{net} = m\vec{a}$$
$$F_{net_x} = ma_x$$
$$T_h - f = ma$$
$$T_h - \mu_k n = ma$$
$$T_h - \tfrac{1}{2}mg = ma$$

We have our three equations. Time for algebra (on the next page).

Long Answer 3 continued

$3mg - T_v = 3ma$

$T_v = 3m(g-a)$

$T_v - T_h = ma$

$T_h - \frac{1}{2}mg = ma$

$T_h = m(\frac{1}{2}g + a)$

$3mg - 3ma - \frac{1}{2}mg - ma = ma$

$\frac{5}{2}g = 5a$

$a = \frac{1}{2}g$

$T_v = 3m(g-a)$

$= 3m(g - \frac{1}{2}g)$

$= \frac{3}{2}mg$

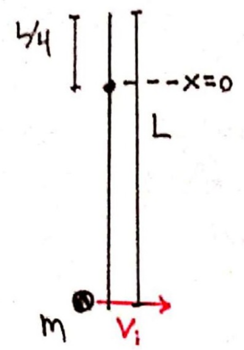
$T_h = m(\frac{1}{2}g + a)$

$= m(\frac{1}{2}g + \frac{1}{2}g)$

$= mg$

$$\boxed{\begin{aligned} a &= \tfrac{1}{2}g \\ T_{vert} &= \tfrac{3}{2}mg \\ T_{hor} &= mg \end{aligned}}$$

Long Answer 4



a) This is much like what we did for the second long answer problem.

$$I = \int_{rod} r^2 dm = \int_{-L/4}^{3L/4} x^2 \lambda dx = \int_{-1m}^{3m} x^2\left[\left(3\tfrac{kg}{m}\right)+\left(15\tfrac{kg}{m^3}\right)x^2\right]dx$$

$$I = \left[\left(1\tfrac{kg}{m}\right)x^3 + \left(3\tfrac{kg}{m^3}\right)x^5\right]\Big|_{x=-1m}^{3m} = \boxed{760\ kg\cdot m^2}$$

b) This is a job for conservation of angular momentum

$$\vec{L}_{putty_i} + \vec{L}_{rod_i} = \vec{L}_{together_f}$$

$$\vec{r}\times\vec{p}_{putty_i} + 0 = (I_{rod} + I_{putty})\vec{\omega_f}$$

$$r m_{putty} V_{putty_i} \sin 90° (-\hat{z}) = \left(I_{rod} + m_{putty} r^2\right)\vec{\omega_f}$$

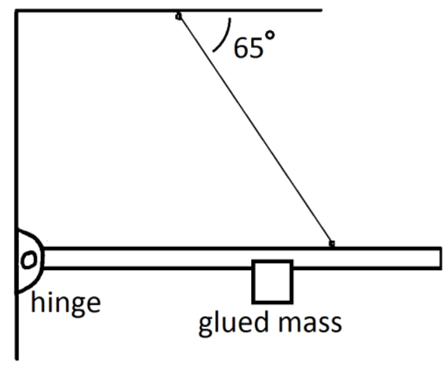
$$\vec{\omega_f} = \frac{\frac{3L}{4} m_{putty} V_{putty_i} (-\hat{z})}{\left(I_{rod} + m_{putty}\left(\frac{3L}{4}\right)^2\right)}$$

$$\boxed{\vec{\omega_f} = 0.0158\ rad/s\ (-\hat{z})}$$ (Into the page)

c) Total mechanical energy is conserved after the collision (gravity is a conservative force). We would set the initial rotational kinetic energy equal to the gravitational potential energy at the maximum displacement from equilibrium, expressing the height of the center of mass as a function of the displacement angle from the vertical.

5:

A long uniform horizontal rod of total length 10.0 m and total mass 150 kg it attached to a wall by a hinge (pivot). It is supported by a wire attached at 7.00 m from the hinge. The wire makes and angle of 65˚ with the ceiling as shown.



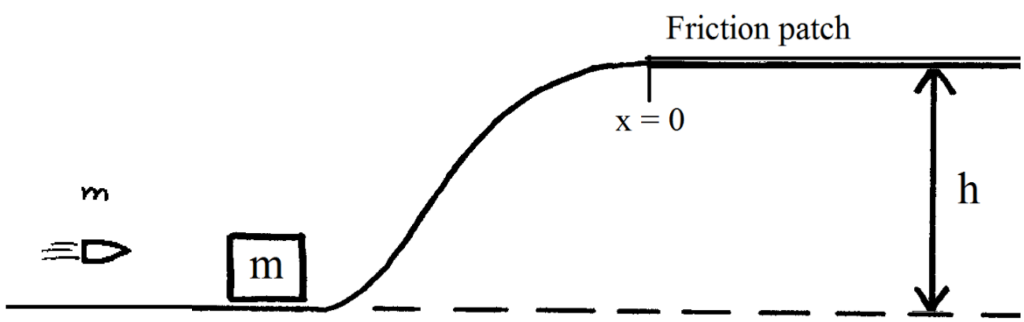
The maximum tension allowed in the wire before it will break is 2016 N. You want to glue a block of mass 60.0 kg on the rod as far away from the hinge as possible.

a) Calculate the maximum distance away from the hinge that you can glue the block before the string breaks.

b) When the block is in this maximum position, the string breaks. The rod/mass combination swings downwards, starting from rest. Find the angular speed of the rod/mass combination just as they reach the vertical position.

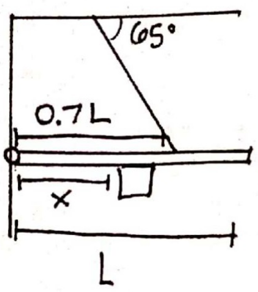
6:

A bullet of mass m is shot with an initial speed v_o into a block of mass M that is initially at rest. It collides and sticks inside of the block. After the collision, the bullet and block combination move up a frictionless hill to a height h above the ground. Once at the top of the hill, there is a rough patch with uniform friction characterized by the kinetic coefficient μ_k.

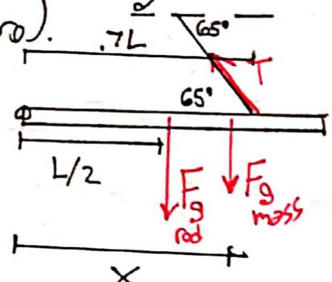


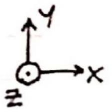
How far does the bullet/block combination travel through the rough patch before coming to rest? Write your answer for this distance z in terms of only the variables m, M, v, h, μ_k and g.

Long Answer 5



When the rod is in static equilibrium, $\vec{\tau}_{net} = 0$.
There are four forces acting on the rod: The rod's weight (acting at its center of mass), the weight of the glued mass (acting at its position), the tension in the string (acting at .7L from the wall), and the force exerted by the hinge. We set our axis of rotation, eliminating the torque from the hinge (since its distance from our axis is zero).





a)

$$\vec{\tau}_{net} = 0$$

$$\vec{\tau}_{hinge} + \vec{\tau}_{\substack{rod\\weight}} + \vec{\tau}_{\substack{mass\\weight}} + \vec{\tau}_T = 0$$

$$0 + \frac{L}{2} m_{rod} g \sin 90°(-\hat{z}) + X m_{mass} g \sin 90°(-\hat{z}) + .7LT \sin 65° = 0$$

$$-\frac{L}{2} m_{rod} g - X m_{mass} g + .7LT \sin 65° = 0$$

We find the maximum safe value for x by setting $T = T_{max}$. If x gets any bigger, the cable will snap.

$$-\frac{L}{2} m_{rod} g - X_{max} m_{mass} g + .7L T_{max} \sin 65° = 0$$

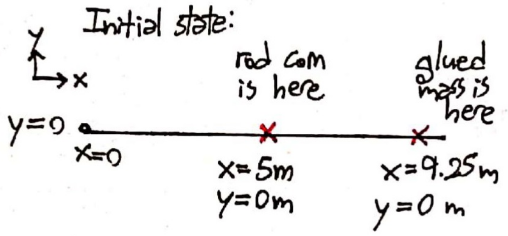
$$X_{max} = \frac{\frac{7}{10} L T_{max} \sin 65° - \frac{1}{2} L m_{rod} g}{m_{mass} g}$$

$$\boxed{X_{max} = 9.25 \text{ m}}$$

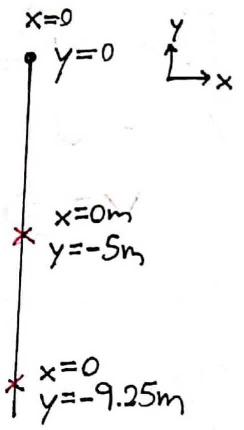
Long Answer 5

b) This is a job for energy conservation. We'll call $y=0$ where the rod was when it was horizontal.

Initial state:



Final state:



$$U_{gi} + K_i = U_{gf} + K_f$$

$$0 + 0 = m_{rod}\, g\, y_{com_f} + m_{mass}\, g\, y_{mass_f} + \frac{1}{2} I \omega_f^2$$

The total moment of inertia is that of a uniform rod pivoted about its end plus that of a point mass (the glued block) pivoted about that same axis.

$$I = \frac{1}{3} m_{rod} L^2 + m_{mass} r^2 = \frac{1}{3}(150\,kg)(10.0\,m)^2 + (60.0\,kg)(9.25\,m)^2$$

$$I = 10133.75\ kg \cdot m^2$$

Now we solve for ω_f:

$$0 = m_{rod}\, g\, y_{com_f} + m g\, y_{mass_f} + \frac{1}{2} I \omega_f^2$$

$$\omega_f^2 = \frac{-2g}{I}\left(m_{rod}\, y_{com_f} + m_{mass}\, y_{mass_f}\right)$$

$$\omega_f^2 = \frac{-2(9.81\,m/s^2)}{10133.75\,kg \cdot m^2}\left[150\,kg \cdot (-5\,m) + 60\,kg \cdot (-9.25\,m)\right]$$

$$\boxed{\omega_f = 1.60\ rad/s}$$

Long Answer 6

Two steps. First, we use conservation of linear momentum to find the speed of the bullet+block system immediately after impact:

$$\vec{p}_{\substack{bullet\\before}} + \vec{p}_{\substack{block\\before}} = \vec{p}_{\substack{together\\after}}$$

$$mV_0 \quad + \quad 0 \quad = (m+M)V_0$$

$$V = \left(\frac{m}{m+M}\right)V_0$$

This speed informs our initial state for step two, in which we apply energy conservation. Our initial state is immediately after the collision, our final state is after the bullet+block system has come to rest, having traveled a distance Z through the friction patch.

$$K_i + U_{gi} = K_f + U_{gf} + \Delta U_{int}$$

$$\frac{1}{2}\cancel{(m+M)}\, V^2 + 0 = 0 + \cancel{(m+M)}\, gh + \cancel{(m+M)}\, g\mu_k Z$$

$$\frac{1}{2}\left(\frac{mV_0}{m+M}\right)^2 = gh + g\mu_k Z$$

$$\frac{1}{2}\left(\frac{mV_0}{m+M}\right)^2 - gh = g\mu_k Z$$

$$\boxed{Z = \frac{\frac{1}{2}\left(\frac{mV_0}{m+M}\right)^2 - gh}{g\mu_k}}$$