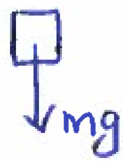
REVIEW FOR EXAM 1

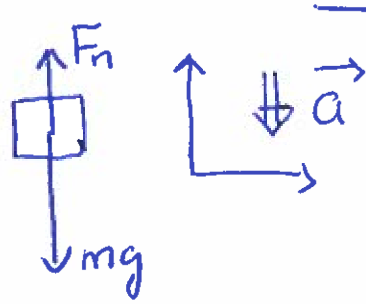
Note: <u>This is not comprehensive</u>. To be prepared for the exam, you must study ALL the required materials. This is meant for you to get an overview of some main topics to help jog your memories about those topics and ask any questions you have.

1. Draw free body diagrams for the following:
   a. An object in projectile motion that was thrown at an angle and is halfway to the apex of its trajectory. Neglect air resistance.

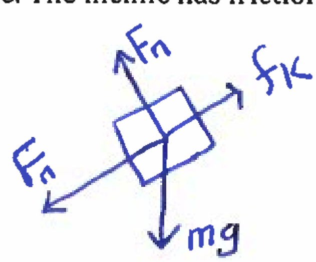


Only mg acts at all points, even at top!

   b. A person in an elevator that is ascending at a decreasing speed.

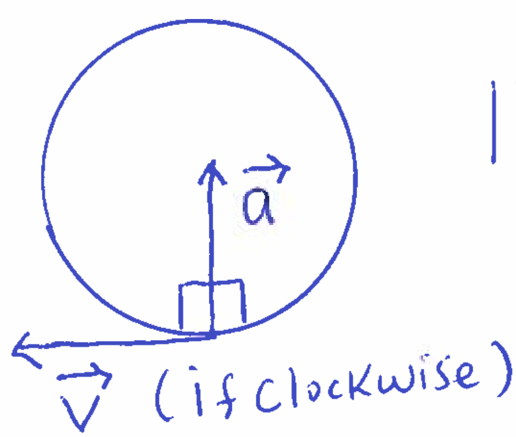


   c. A block being pushed down an incline by a force F that acts parallel to the incline. The incline has friction.



Unknown $\vec{a}$ direction so draw anything you want in terms of relative vectors.

2. Consider a cart of mass m that is going around a loop-the-loop of radius R. Ignore friction. Assume it is a constant speed v. When it is at the very bottom of the loop, describe the acceleration and velocity vectors by drawing a diagram. Also state the magnitude of the acceleration.



$$|\vec{a}| = \frac{v^2}{R}$$

3. An object has no net force acting on it. Is it moving? Explain your answer.

It might be! $\sum \vec{F} = 0$ means $\vec{a} = 0$ NOT $\vec{v} = 0$

Just means $\vec{v}$ is not changing.

4. How many different types of energies do we use in our conservation of energy problems? List the types by name and symbol

Four types:

Initial $[K, U_g, U_{el}]$

Final $[K, U_g, U_{el}, \Delta U_{int}]$

5. Is the change in internal energy ever included in the initial energy?

No! Only in final

6. How does the change in internal energy relate to the work done by non-conservative forces?

$$\Delta U_{int} = - W_{non,cons}$$

7. What makes a force non-conservative? Give an example.

Work done is path dependent

e.g. friction.

8. Does the absolute gravitational potential energy ever matter? If not, what does matter?

No. Ug depends on choice of coordinates (where $y = 0$). But ΔU_g matters $\Rightarrow$ Everyone agrees on ~~this~~.

9. How do you get force from a potential energy?

$$F_x = -\frac{dU}{dx}$$

Note Negative!

10.

a) What *is* work? Explain in words.

Amount of energy transfered

b) What is the unit for work?

Joule (like energy). ($J = N \cdot m$)

c) What is the general equation for finding work from a force?

$W = \int \vec{F} \cdot d\vec{r}$ Scalar/dot Product

d) How does this simplify if you have a constant force?

$W = \vec{F} \cdot \Delta\vec{r}$ (No Integral)

e) How does this simplify if the force is always perpendicular to the displacement/velocity?

$W = \vec{F} \cdot \Delta\vec{r} = F \Delta r \cos\theta, \ \theta = 90^\circ$

$= 0$

f) How does this simplify if you have a non-constant but 1D force?

$W = \int F_x \, dx \leftarrow$ No scalar product

g) If you have a non-constant 1D force, how can you get work from a graph? What graph do you need?

Area under F_x vs x

11. What is power, in words, what is the unit of power?

$$\text{Power} = \frac{\text{Work done}}{\text{time}}$$

12. What is the Work-Kinetic Energy theorem?

$W = K_f - K_i = \Delta K$

13. You are given a graph of Force versus position. What information can you obtain from this graph?

$\underbrace{F \text{ vs. } x}_{\text{Area under.}}$ gives work $= \Delta K$

14. Say an object has:

$$v(t) = (-20 + 5.0\,t^2, 2.0t^4) \text{ m/s}$$

and starts at r = (3, 0) meters at t = 0.

a. Find the acceleration at t = 2 seconds.

$$\vec{a}(t) = \frac{d\vec{v}}{dt} = (10t, 8t^3)\ m/s^2$$

$$a(t=2s) = (20, 64)\ m/s^2$$

b. Find the time when the speed along the x direction is zero.

Set $V_x = 0$ and find time.

$$5.0t^2 - 20 = 0$$
$$t^2 = 4$$
$$t = 2.0 \text{ sec.}$$

c. Find the position at t = 2 seconds.

$$\vec{r}(t) = \int V(t)dt = \left(\int(-20+5t^2)dt, \int(2t^4)dt\right)$$

$$\vec{r}(t) = \left(-20t + \frac{5}{3}t^3 + 3,\ \frac{2}{5}t^5\right)$$

Plug in $t = 2s$

$$\vec{r}(t=2) = (-23.7, 12.8)\ m$$

15. A ball is thrown vertically upwards near the surface of the Earth. It has an initial speed v_o at t = 0. At what **times** does it have a speed equal to $v_o/2$?

Given
$a = -g$
$V_o = V_o$
$V_f = V_o/2$ or $(-V_o/2)$
Want t for each.

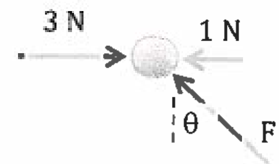
$V_f = V_o + at \Rightarrow t = \frac{V_f - V_o}{a}$

If $V_f = V_o/2$, $t = \frac{\frac{V_o}{2} - V_o}{-g} = \frac{V_o}{2g}$ on way up

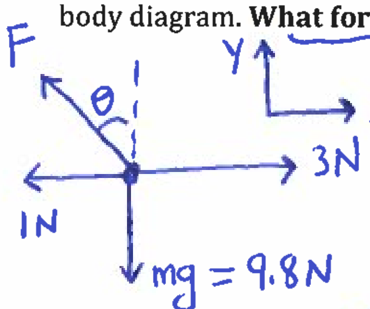
If $V_f = -V_o/2$

$t = \frac{-V_o/2 - V_o}{-g} = \frac{3V_o}{2g}$ ✓ on way down

16. The object shown has a mass m = 1 kg and is being suspended in the air (near the surface of the Earth) by three people pushing on it as shown. No net force acts on it. Find the value of F and the angle θ. (HINT: This is NOT the complete free body diagram. **What force is missing?**)



$\to mg$



$$\sum F_x = 3N - 1N - F\sin\theta = 0$$

$$F\sin\theta = 2N$$

$$\sum F_y = 0$$

$$F\cos\theta - 9.8N = 0$$

$$F\cos\theta = 9.8N \rightarrow F = \frac{9.8N}{\cos\theta}$$

$$\frac{9.8N}{\cos\theta}\sin\theta = 2N, \quad \tan\theta = \frac{2}{9.8} \rightarrow \theta = \tan^{-1}\left(\frac{2}{9.8}\right) = 11.5°$$

$$F = \frac{9.8\ N}{\cos(11.5°)} = 10N$$

17. A plane flies at a level trajectory at a height h above the ground, which is also level. It has a constant speed v when it releases a package. What horizontal distance does the package travel relative to the release point before it strikes the ground below? Show all work, and put your answer in terms of *h*, *v*, and *g*, where g is positive (as it always is!).

X

$V_x = V$

Want $D = V_x t$

(Find t from Y)

$$\boxed{D = V\sqrt{\frac{2h}{g}}}$$

Y

$V_{y_0} = 0$

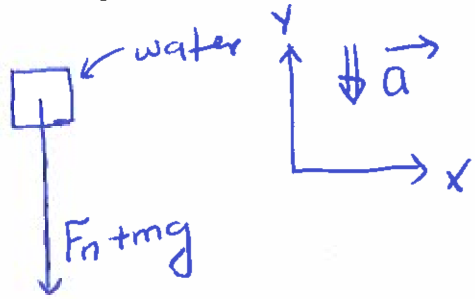
$Y_f = 0, Y_0 = h$

$$\underbrace{Y_f}_{=0} = \underbrace{Y_0}_{=h} + \underbrace{V_{y_0}}_{=0} t - \frac{1}{2}gt^2$$

$$-h = -\frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2h}{g}}$$

18. A pail of water is spun in a vertical circle of radius R. If spun too slowly, the water will fall out at the top. What minimum speed must the pail and water have at the top in order for the water to stay in the pail? The water's mass is m. How can you use this to find the minimum required speed of the pail at any give point in the revolution? Explain in words



$$\sum F_y = m a_y$$

$$-F_n - mg = -m\frac{v^2}{R}$$

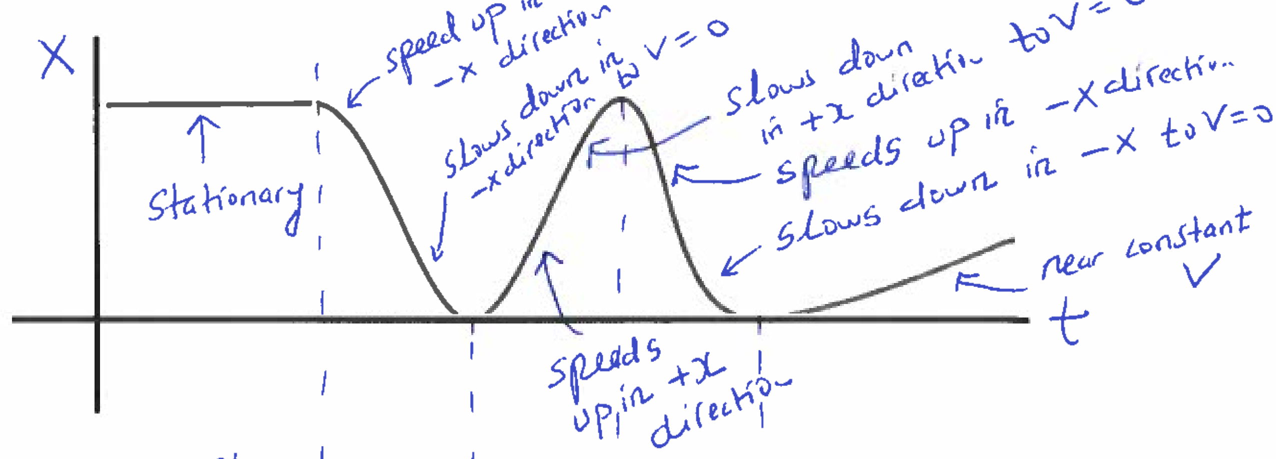
$$F_n \to 0,\quad V \to V_{min,\,top}$$

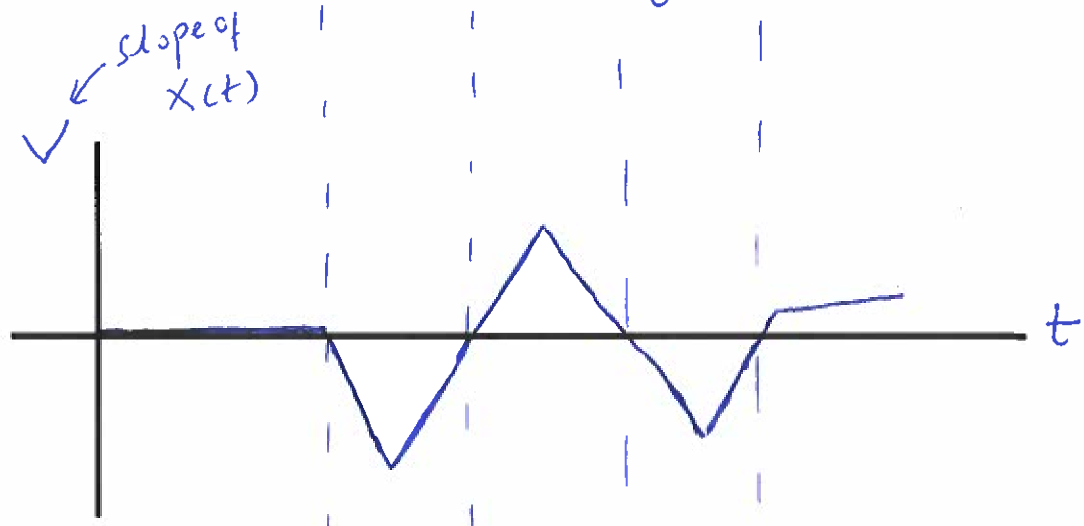
$$\cancel{m}g = \frac{\cancel{m}V_{min,\,top}^2}{R}$$

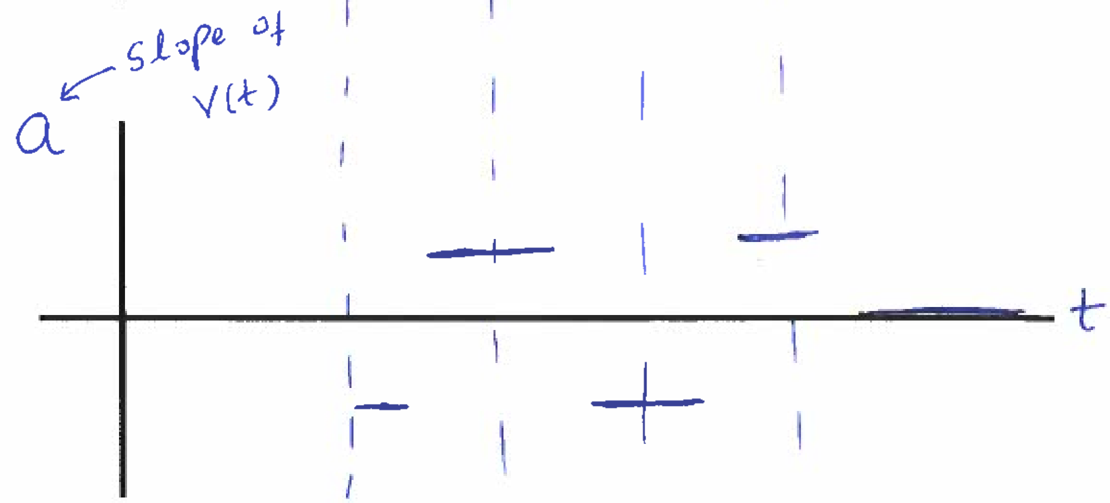
$$V_{min,\,top} = \sqrt{gR}$$

You can use ~~this~~ to find the speed at any other point in the rotation by using conservation of energy, where KE at the top is $\left(\frac{1}{2} mRg\right)$ and the potential energy will be $mg2R$ the relative to the bottom of the circle.

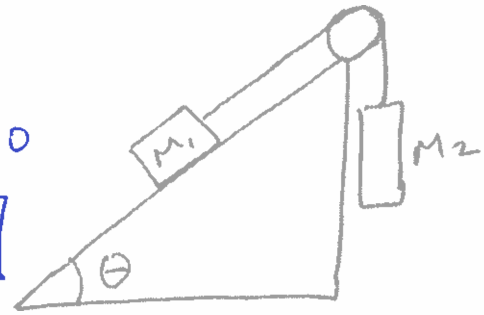
19. Consider the graph shown. In words, describe the motion, and draw the corresponding velocity versus time and acceleration versus time graphs below using the same time scale. **Assume all curved portions shown on the x (t) graph are parabolic.**



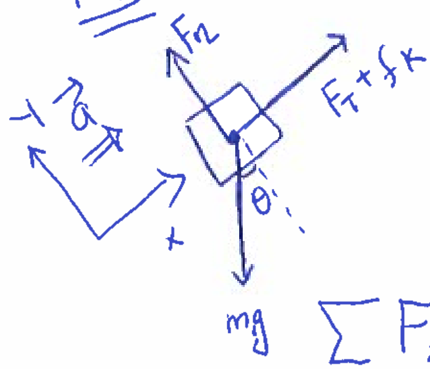




20. Assume the ramp has a coefficient of kinetic friction μ_k. You are told that block m_1 is originally headed down the ramp (because someone pushed it) but slowing down. Write the two equations that you would use to solve for the magnitudes of T and a **after the push** given μ_k, m_1, m_2, g, and θ. Show all work, including clear free body diagrams. *Do not solve for T and a… just know that you can.*



m_1



$\sum F_y = 0$

$F_n - m_1 g\cos\theta = 0$

$\boxed{F_n = m_1 g\cos\theta}$

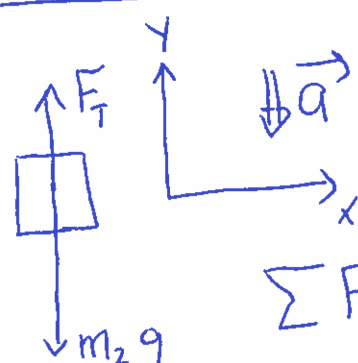
$\sum F_x = m_1 a_x$

$F_T + f_K - m_1 g\sin\theta = m_1 a$

$f_K = \mu_K F_n = \mu_K m_1 g\cos\theta$

$\boxed{F_T + \mu_K m_1 g\cos\theta - m_1 g\sin\theta = m_1 a}$ (1)

m_2



$\sum F_y = m_2 a_y$

$F_T - m_2 g = -m_2 a$

$\boxed{F_T - m_2 g = -m_2 a}$ (2)

Using (1) and (2) could solve for F_T and a.

21. A block of mass m approaches the foot of a hill with a speed of v_0. It heads up the frictionless hill of height h and reaches the top. The top is rough with coefficient of kinetic friction μ_k over a distance d. After the rough patch, there is a spring of spring constant k. Ignore air resistance.

a. Assume that the block does not make it through the rough patch. Determine the length L that it travels through the rough patch before coming to rest.

Initial

$K_0 = \frac{1}{2} m V_0^2$

Final

$U_{gf} = mgh$; $\Delta U_{int} = \mu_K F_n L$

Find

$F_n = mg$

So $\frac{1}{2} \cancel{m} V_0^2 = \cancel{m}gh + \mu_K \cancel{m} g L$

$$L = \frac{V_0^2}{2\mu_K g} - \frac{h}{\mu_K}$$

b. For the situation in (a), what is the work done by non-conservative forces?

$$W = -\Delta U_{int} = -\mu_K m g L$$

where

$$L = \frac{V_0^2}{2\mu_K g} - \frac{h}{\mu_K}$$

c. Now assume that the block does make it through the rough patch. Determine the maximum compression of the spring.

Initial

$K_0 = \frac{1}{2} m V_0^2$

Final

$U_{gf} = mgh$; $\Delta U_{int} = \mu_K mgd$

$U_{el} = \frac{1}{2} K X^2$

$$\frac{1}{2} m V_0^2 = mgh + \mu_K mgd + \frac{1}{2} K x^2$$

↳ Solve for x

$$x = \sqrt{\frac{m}{K}\left(V_0^2 - 2gh - 2\mu_K g d\right)}$$

22. A mass *m* is pushed against a spring with spring constant *k* and held in place with a catch. The spring compresses an unknown distance *x*. When the catch is removed, the mass leaves the spring and slides along a frictionless circular loop of radius *r*. When the mass reaches the top of the loop, the force of the loop on the mass (the normal force) is equal to twice the weight of the mass.

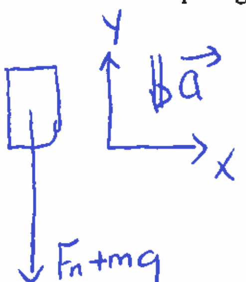
a) Using conservation of energy, find the kinetic energy at the top of the loop. Express your answer as a function of k, m, x, g, and R.

$$\frac{1}{2}Kx^2 = K_f + mg(2R)$$

$$K_f = \frac{1}{2}Kx^2 - 2mgR \quad (1)$$

b) How far was the spring compressed?

Free body diagram at top



$$\sum F_y = ma_y$$

$$-F_n - mg = -\frac{mV_f^2}{R}$$

$$-2mg - mg = -\frac{mV_f^2}{R}$$

$$mV_f^2 = 3mgR \quad (2)$$

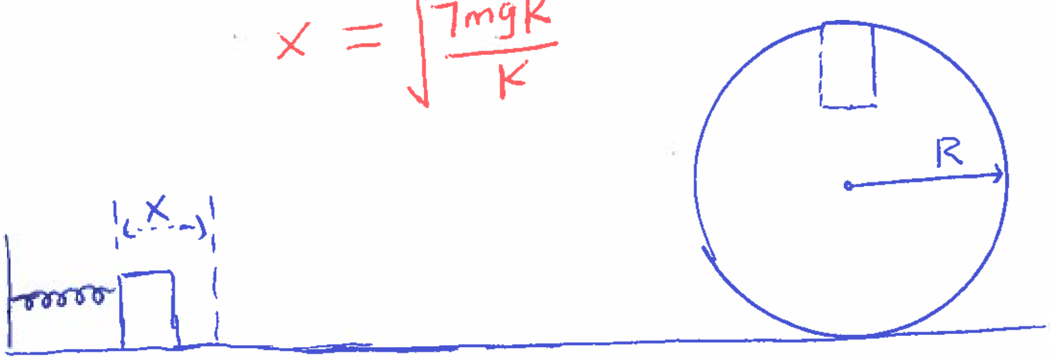
Substitute (2) in (1)

$$K_f = \frac{1}{2}mV_f^2 = \frac{3}{2}mgR$$

$$\frac{3}{2}mgR = \frac{1}{2}Kx^2 - 2mgR$$

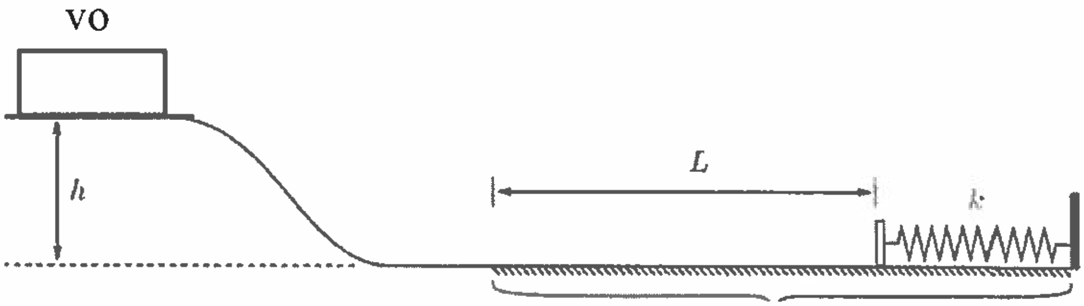
$$\frac{1}{2}Kx^2 = \frac{7}{2}mgR$$

$$x = \sqrt{\frac{7mgR}{K}}$$



$$E_0 = U_{el} = \frac{1}{2}Kx^2$$

23. A block of mass m=8.00 kg moves with initial speed vo=6.00 m/s along a track, as shown. The upper part of the track is a height h=10.0 m above the lower part of the track. There is no friction between the block and the track until the block reaches the rough patch on the lower part of the track, at which point the coefficients of kinetic and static friction are μk=0.250 and μs=0.350 respectively. After traveling a distance L=16.0 mover the rough patch, the block collides with a very long, ideal spring with spring constant k. The block compresses the spring by a maximum distance d = 4.00 m, *with friction still acting upon the block.* Calculate the spring constant.



Energy Conservation

Initial

$\frac{1}{2} m V_0^2 + mgh$

Final

$\frac{1}{2} K d^2 + \mu_K mg(L+d)$

So

$$\frac{1}{2} m V_0^2 + mgh = \frac{1}{2} K d^2 + \mu_K mg (L+d)$$

$$\frac{1}{2} K d^2 = \frac{1}{2} m V_0^2 + mgh - \mu_K mg (L+d)$$

$$K = \frac{2}{d^2} \left(\frac{1}{2} m V_0^2 + mgh - \mu_K mg (L+d) \right)$$

$$\boxed{K = 67 \text{ N/m}}$$

GOOD LUCK!!! **STUDY HARD!!!** **YOU CAN DO IT!!!**