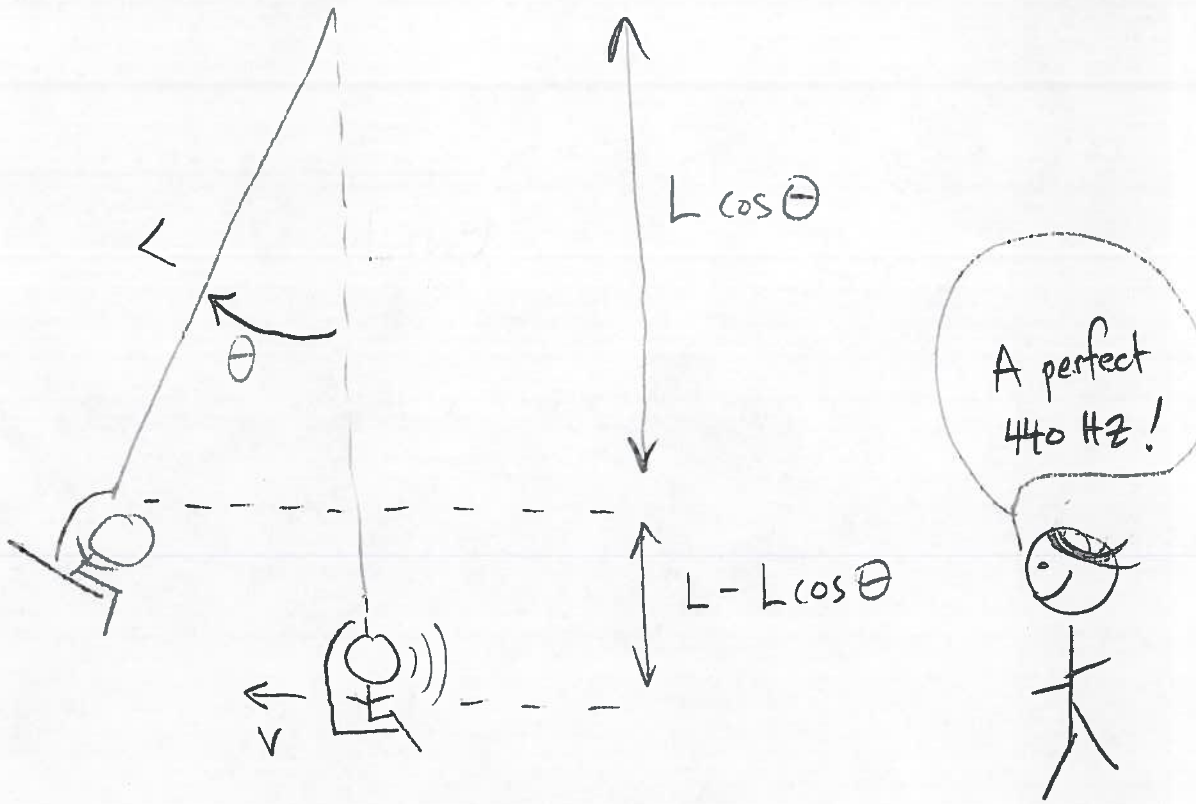

Justin swings on a swingset with rope of length L, at an angle of at most $\theta = 10°$. When he swings at max speed away from his audience, she hears a perfect $f' = 440$ Hz ... even though Justin produces $f = 450$ Hz.

a) Justin is at the bottom of his swing when the speed and Doppler shift are a maximum.

b) How fast is Justin moving?

$$f' = f\left(\frac{1}{1 + \frac{V}{V_s}}\right)$$

V = Justin's speed
V_s = sound speed = 343 m/s

$$1 + \frac{V}{V_s} = \frac{f}{f'}$$

$$V = V_s\left[\frac{f}{f'} - 1\right]$$

So Justin's maximum speed is

$$V = \left(343\ \tfrac{m}{s}\right)\left[\frac{450\ Hz}{440\ Hz} - 1\right] = 7.8\ \tfrac{m}{s}$$

From diagram at top of previous page, we see that Justin's height at top of a swing is

$$h = L - L\cos\theta$$

But conservation of energy requires

$$mgh = \tfrac{1}{2}mv^2$$

$$\Rightarrow V^2 = 2gh = 2g\left[L - L\cos\theta\right]$$

$$V = \sqrt{2gL(1-\cos\theta)}$$

Re-arrange to solve for L

$$\frac{V^2}{2g(1-\cos\theta)} = L$$

$$L = \frac{(7.8\ m/s)^2}{2(9.8\ m/s^2)(1-\cos 10^\circ)} = 204\ m\ !$$

That's one big swingset ! The period of the motion will be

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{g}{L}}} = 2\pi\sqrt{\frac{L}{g}} = 28.7\ s$$