$$L = L_0 + 0.9856474(t - t_0)$$

$$g = g_0 + 0.9856003(t - t_0)$$

$$\lambda = L + 1.915\sin(g) + 0.020\sin(2g)$$

$$(\alpha - \alpha_0) = \varpi\big[\cos(\alpha)\cos(\epsilon)\sin(\lambda) - \sin(\alpha)\cos(\lambda)\big] + \mu_\alpha t + K$$

$$\cos(\alpha_0) \simeq \cos(\alpha)$$

$$\sin(\alpha_0) \simeq \sin(\alpha)$$

$$\cos(\delta_0) \simeq \cos(\delta)$$

$$\sin(\delta_0) \simeq \sin(\delta)$$

$$(\alpha - \alpha_0) \simeq \varpi\big[\cos(\alpha_0)\cos(\epsilon)\sin(\lambda) - \sin(\alpha_0)\cos(\lambda)\big] + \mu_\alpha t + K$$

$$P \equiv \cos(\alpha_0)\cos(\epsilon)\sin(\lambda) - \sin(\alpha_0)\cos(\lambda)$$

$$(\alpha - \alpha_0) \simeq \varpi P + \mu_\alpha t + K$$

$$\begin{aligned}(\delta - \delta_0) = \varpi\big[&\cos(\delta)\sin(\epsilon)\sin(\lambda) - \cos(\alpha)\sin(\delta)\cos(\lambda) \\ &- \sin(\alpha)\sin(\delta)\cos(\epsilon)\sin(\lambda)\big] + \mu_\delta t + L\end{aligned}$$

$$\begin{aligned}(\delta - \delta_0) = \varpi\big[&\cos(\delta_0)\sin(\epsilon)\sin(\lambda) - \cos(\alpha_0)\sin(\delta_0)\cos(\lambda) \\ &- \sin(\alpha_0)\sin(\delta_0)\cos(\epsilon)\sin(\lambda)\big] + \mu_\delta t + L\end{aligned}$$

$$\begin{aligned}Q \equiv\ &\cos(\delta_0)\sin(\epsilon)\sin(\lambda) - \cos(\alpha_0)\sin(\delta_0)\cos(\lambda) \\ &- \sin(\alpha_0)\sin(\delta_0)\cos(\epsilon)\sin(\lambda)\end{aligned}$$

$$(\delta - \delta_0) = \varpi Q + \mu_\delta t + L$$

$$\beta = \sum \left[\Delta\mathrm{RA}_i - (\alpha - \alpha_0)\right]^2 + \left[\Delta\mathrm{Dec}_i - (\delta - \delta_0)\right]^2$$

$$\beta = \sum \left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right]^2 + \left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right]^2$$

$$\begin{aligned}\frac{\partial\beta}{\partial\varpi} &= \sum (-2P_i)\left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right] + \\ &\quad (-2Q_i)\left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right]\\ \frac{\partial\beta}{\partial\varpi} &= 0\end{aligned}$$

$$\begin{aligned}0 &= \sum \Delta\mathrm{RA}_i P_i - \sum \varpi P_i^2 - \sum \mu_\alpha P_i t_i - \sum K P_i + \\ &\quad \sum \Delta\mathrm{Dec}_i Q_i - \sum \varpi Q_i^2 - \sum \mu_\delta Q_i t_i - \sum L Q_i\end{aligned}$$

$$\begin{aligned}\varpi \sum \left(P_i^2 + Q_i^2\right) + \mu_\alpha \sum P_i t_i + \mu_\delta \sum Q_i t_i + K \sum P_i + L \sum Q_i \\ = \sum \left(\Delta\mathrm{RA}_i P_i + \Delta\mathrm{Dec}_i Q_i\right)\end{aligned} \tag{1}$$

$$\frac{\partial\beta}{\partial\mu_\alpha} = \sum (-2t_i)\left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right]$$

$$0 = \sum \Delta\mathrm{RA}_i t_i - \sum \varpi P_i t_i - \sum \mu_\alpha t_i^2 - \sum K t_i$$

$$\varpi \sum P_i t_i + \mu_\alpha \sum t_i^2 + K \sum t_i = \sum \Delta\mathrm{RA}_i t_i$$

$$\varpi \sum P_i t_i + \mu_\alpha \sum t_i^2 + \mu_\delta\, 0 + K \sum t_i + L\, 0 = \sum \Delta\mathrm{RA}_i t_i \tag{2}$$

$$\frac{\partial\beta}{\partial\mu_\delta} = \sum (-2t_i)\left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right]$$

$$0 = \sum \Delta\mathrm{Dec}_i t_i - \sum \varpi Q_i t_i - \sum \mu_\delta t_i^2 - \sum L t_i$$

$$\varpi \sum Q_i t_i + \mu_\delta \sum t_i^2 + L \sum t_i = \sum \Delta\mathrm{Dec}_i t_i$$

$$\varpi \sum Q_i t_i + \mu_\alpha\, 0 + \mu_\delta \sum t_i^2 + K\, 0 + L \sum t_i = \sum \Delta\mathrm{Dec}_i t_i \tag{3}$$

$$
\begin{aligned}
\frac{\partial \beta}{\partial K} &= \sum (-2)\ \left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right] \\
0 &= \sum \Delta\mathrm{RA}_i \ - \ \sum \varpi P_i \ - \ \sum \mu_\alpha t_i \ - \ \sum K \\
\varpi \sum P_i \ + \ \mu_\alpha \sum t_i \ + \ K \sum 1 &= \sum \Delta\mathrm{RA}_i
\end{aligned}
$$

$$
\varpi \sum P_i \ + \ \mu_\alpha \sum t_i \ + \ \mu_\delta\, 0 \ + \ K \sum 1 \ + \ L\, 0 \quad = \quad \sum \Delta\mathrm{RA}_i \tag{4}
$$

$$
\begin{aligned}
\frac{\partial \beta}{\partial L} &= \sum (-2)\ \left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right] \\
0 &= \sum \Delta\mathrm{Dec}_i \ - \ \sum \varpi Q_i \ - \ \sum \mu_\delta t_i \ - \ \sum L \\
\varpi \sum Q_i \ + \ \mu_\delta \sum t_i \ + \ L \sum 1 &= \sum \Delta\mathrm{Dec}_i
\end{aligned}
$$

$$
\varpi \sum Q_i \ + \ \mu_\alpha\, 0 \ + \ \mu_\delta \sum t_i \ + \ K\, 0 \ + \ L \sum 1 \quad = \quad \sum \Delta\mathrm{Dec}_i \tag{5}
$$

$$
\begin{pmatrix}
\sum \left(P_i^2 + Q_i^2\right) & \sum P_i t_i & \sum Q_i t_i & \sum P_i & \sum Q_i \\
\sum P_i t_i & \sum t_i^2 & 0 & \sum t_i & 0 \\
\sum Q_i t_i & 0 & \sum t_i^2 & 0 & \sum t_i \\
\sum P_i & \sum t_i & 0 & \sum 1 & 0 \\
\sum Q_i & 0 & \sum t_i & 0 & \sum 1
\end{pmatrix}
\begin{pmatrix}
\varpi \\ \mu_\alpha \\ \mu_\delta \\ K \\ L
\end{pmatrix}
=
\begin{pmatrix}
\sum \left(\Delta\mathrm{RA}_i P_i \ + \ \Delta\mathrm{Dec}_i Q_i\right) \\
\sum \Delta\mathrm{RA}_i t_i \\
\sum \Delta\mathrm{Dec}_i t_i \\
\sum \Delta\mathrm{RA}_i \\
\sum \Delta\mathrm{Dec}_i
\end{pmatrix}
$$

$$
\begin{aligned}
\mathbf{A} &\equiv
\begin{pmatrix}
\sum \left(P_i^2 + Q_i^2\right) & \sum P_i t_i & \sum Q_i t_i & \sum P_i & \sum Q_i \\
\sum P_i t_i & \sum t_i^2 & 0 & \sum t_i & 0 \\
\sum Q_i t_i & 0 & \sum t_i^2 & 0 & \sum t_i \\
\sum P_i & \sum t_i & 0 & \sum 1 & 0 \\
\sum Q_i & 0 & \sum t_i & 0 & \sum 1
\end{pmatrix} \\
\mathbf{x} &\equiv
\begin{pmatrix}
\varpi \\ \mu_\alpha \\ \mu_\delta \\ K \\ L
\end{pmatrix} \\
\mathbf{b} &\equiv
\begin{pmatrix}
\sum \left(\Delta\mathrm{RA}_i P_i \ + \ \Delta\mathrm{Dec}_i Q_i\right) \\
\sum \Delta\mathrm{RA}_i t_i \\
\sum \Delta\mathrm{Dec}_i t_i \\
\sum \Delta\mathrm{RA}_i \\
\sum \Delta\mathrm{Dec}_i
\end{pmatrix}
\end{aligned}
$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\begin{aligned}
w_{\mathrm{RA},i} &\equiv \frac{1}{\sigma_{\mathrm{RA}_i}} \\
w_{\mathrm{Dec},i} &\equiv \frac{1}{\sigma_{\mathrm{Dec}_i}}
\end{aligned}$$

$$\beta = \sum \left[\Delta\mathrm{RA}_i - (\alpha - \alpha_0)\right]^2 w_{\mathrm{RA},i}^2 + \left[\Delta\mathrm{Dec}_i - (\delta - \delta_0)\right]^2 w_{\mathrm{Dec},i}^2$$

$$\begin{aligned}
\beta &= \sum \left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right]^2 w_{\mathrm{RA},i}^2 + \\
&\quad \left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right]^2 w_{\mathrm{Dec},i}^2 \\
\frac{\partial\beta}{\partial\varpi} &= \sum (-2P_i) \left[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\right] w_{\mathrm{RA},i}^2 + \\
&\quad (-2Q_i) \left[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\right] w_{\mathrm{Dec},i}^2
\end{aligned}$$

$$\begin{aligned}
&\varpi \sum \left(P_i^2\, w_{\mathrm{RA},i}^2 + Q_i^2\, w_{\mathrm{Dec},i}^2\right) + \mu_\alpha \sum P_i t_i\, w_{\mathrm{RA},i}^2 + \\
&\quad \mu_\delta \sum Q_i t_i\, w_{\mathrm{Dec},i}^2 + K \sum P_i\, w_{\mathrm{RA},i}^2 + L \sum Q_i\, w_{\mathrm{Dec},i}^2 \\
&\qquad = \sum \left(\Delta\mathrm{RA}_i P_i\, w_{\mathrm{RA},i}^2 + \Delta\mathrm{Dec}_i Q_i\, w_{\mathrm{Dec},i}^2\right)
\end{aligned}$$

$$\mathbf{A} \equiv \begin{pmatrix}
\sum \left(P_i^2\, w_{\mathrm{RA},i}^2 + Q_i^2\, w_{\mathrm{Dec},i}^2\right) & \sum P_i t_i\, w_{\mathrm{RA},i}^2 & \sum Q_i t_i\, w_{\mathrm{Dec},i}^2 & \sum P_i\, w_{\mathrm{RA},i}^2 & \sum Q_i\, w_{\mathrm{Dec},i}^2 \\
\sum P_i t_i\, w_{\mathrm{RA},i}^2 & \sum t_i^2\, w_{\mathrm{RA},i}^2 & 0 & \sum t_i\, w_{\mathrm{RA},i}^2 & 0 \\
\sum Q_i t_i\, w_{\mathrm{Dec},i}^2 & 0 & \sum t_i^2\, w_{\mathrm{Dec},i}^2 & 0 & \sum t_i\, w_{\mathrm{Dec},i}^2 \\
\sum P_i\, w_{\mathrm{RA},i}^2 & \sum t_i\, w_{\mathrm{RA},i}^2 & 0 & \sum w_{\mathrm{RA},i}^2 & 0 \\
\sum Q_i\, w_{\mathrm{Dec},i}^2 & 0 & \sum t_i\, w_{\mathrm{Dec},i}^2 & 0 & \sum w_{\mathrm{Dec},i}^2
\end{pmatrix}$$

$$\mathbf{x} \equiv \begin{pmatrix} \varpi \\ \mu\alpha \\ \mu_\delta \\ K \\ L \end{pmatrix}$$

$$\mathbf{b} \equiv \begin{pmatrix}
\sum \left(\Delta\mathrm{RA}_i P_i\, w_{\mathrm{RA},i}^2 + \Delta\mathrm{Dec}_i Q_i\, w_{\mathrm{Dec},i}^2\right) \\
\sum \Delta\mathrm{RA}_i t_i\, w_{\mathrm{RA},i}^2 \\
\sum \Delta\mathrm{Dec}_i t_i\, w_{\mathrm{Dec},i}^2 \\
\sum \Delta\mathrm{RA}_i\, w_{\mathrm{RA},i}^2 \\
\sum \Delta\mathrm{Dec}_i\, w_{\mathrm{Dec},i}^2
\end{pmatrix}$$

$$\mathbf{Ax} = \mathbf{b}$$

$$\beta = \sum \Big[\frac{\Delta\mathrm{RA}_i - (\alpha - \alpha_0)}{\sigma_{\mathrm{RA}_i}}\Big]^2 + \Big[\frac{\Delta\mathrm{Dec}_i - (\delta - \delta_0)}{\sigma_{\mathrm{Dec}_i}}\Big]^2$$

$$\beta = \sum \big[\Delta\mathrm{RA}_i - (\varpi P_i - \mu_\alpha t_i - K)\big]^2 + \big[\Delta\mathrm{Dec}_i - (\varpi Q_i - \mu_\delta t_i - L)\big]^2$$

$$\frac{\partial\beta}{\partial\varpi} = 0$$

$$\frac{\partial\beta}{\partial\varpi} = 0$$

$$\frac{\partial\beta}{\partial\varpi} = 0$$

$$\frac{\partial\beta}{\partial\mu_\alpha} = 0$$

$$\frac{\partial\beta}{\partial K} = 0$$

$$\frac{\partial\beta}{\partial\mu_\delta} = 0$$

$$\frac{\partial\beta}{\partial L} = 0$$

$$\mathbf{A}\mathbf{x} = \mathbf{b}$$

$$\mathbf{x} = \mathbf{A}^{-1}\,\mathbf{b}$$

$$\mathrm{PE} = a + bx + cx^2$$