Problem 1:

A photon has an energy of **E = 10.2 eV**.

3 a. What is its wavelength?
3 b. In which part of the electromagnetic spectrum does it belong?
2 c. This photon was emitted by a hydrogen atom. Describe the electronic energy levels involved in the transition which created it.
2 d. What is the minimum wavelength of a photon which can be emitted by a neutral hydrogen atom?

a) $\lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{10.2 \text{ eV}} = 121.6 \text{ nm}$

b) ultraviolet

c) $E_n = \frac{-13.6 \text{ eV}}{n^2}$

$E_1 = -13.6 \text{ eV}$
$E_2 = -3.4 \text{ eV}$

$\Delta E = 10.2 \text{ eV}$

transition $n=2 \rightarrow n=1$

d) max energy drop is $n \cong \infty \rightarrow n=1$

$E_\infty \cong 0 \rightarrow E_1 = -13.6 \text{ eV}$

$\Delta E = 13.6 \text{ eV}$

$\Rightarrow \min \lambda = \frac{1240 \text{ eV}\cdot\text{nm}}{13.6 \text{ eV}} = 91.2 \text{ nm}$

Problem 2:

The cross section for interaction between protons and neutrinos which have energy **1 MeV** is roughly $\sigma = 10^{-50}$ **m²**.

Astronomers in the future construct a neutrino detector by building a giant cube, exactly **L = 3000 m** on each side, and using collisions between the atmosphere inside the cube and incoming gamma rays to create Cerenkov radiation. Treat the atmosphere as pure nitrogen with a volume density of ρ = **1 kg/m⁻³**.

2 a. What is the number density of nitrogen atoms per cubic meter?
2 b. What is the total number of nitrogen atoms inside the cube?
2 c. What is the total cross section for proton-neutrino collisions inside the cube?

a) $m_N = 7p + 7n \cong 14\, m_p = 14 \times 1.67\times10^{-27}\ \text{kg} = 2.34\times10^{-26}\ \text{kg}$

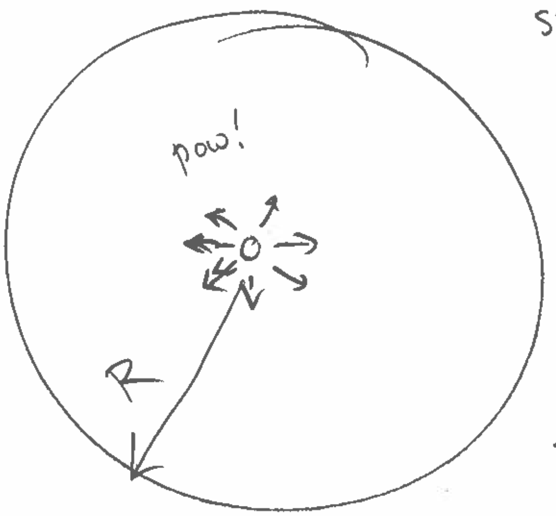
$n_N = \frac{\rho}{m_N} = \frac{1\ \text{kg/m}^3}{2.34\times10^{-26}\ \text{kg}} = 4.28\times10^{25}\ \frac{\text{Nitrogen}}{\text{m}^3}$

b) tot # atoms $= (n_N)(\text{volume}) = 4.28\times10^{25}\ \frac{\text{atoms}}{\text{m}^3} * (3000\text{m})^3 = 1.16\times10^{36}$ atoms

c) each atom has 7 p^+ $\rightarrow \sigma_{tot} = 7 * 1.16\times10^{36}\ \text{atoms} * 10^{-50}\ \text{m}^2 = 8.1\times10^{-14}\ \text{m}^2$

A supernova explodes at the center of our Milky Way, a distance of 8000 pc from Earth. It creates **N = 6 x 10⁵⁷** neutrinos of energy **1 MeV,** which fly outward isotropically from the explosion.

2 d. What is the flux of neutrinos reaching the Earth?
2 e. How many neutrinos will collide with nitrogen atoms inside the cube?



sphere area $A = 4\pi R^2 = 4\pi\left(8000\,\text{pc} * 3.08\times10^{16}\frac{\text{m}}{\text{pc}}\right)^2$
$= 7.63\times10^{41}\ \text{m}^2$

flux $= \frac{\#\text{neutrinos}}{\text{area } A} = \frac{6\times10^{57}\ \text{neutrinos}}{7.63\times10^{41}\ \text{m}^2}$
$= 7.86\times10^{15}\ \frac{\text{neutrinos}}{\text{m}^2}$

#collisions = (flux)(total cross section)
$= f\cdot\sigma_{tot} = 7.86\times10^{15}\ \frac{\nu}{\text{m}^2} * 8.1\times10^{-14}\ \text{m}^2$

$\cong 637$ collisions

Problem 3:

An old star's stellar wind emits one blob of pure helium gas which slams into the surrounding interstellar medium at a speed of **v = 600 km/sec**. The collision creates a strong shock at the blob-ISM interface.

3 a. How much kinetic energy does one atom of helium have before it strikes the ISM?
1 b. How much of that kinetic energy does it lose in the shock?
1 c. What is the shortest wavelength of photon which might be created due to this loss of kinetic energy?
3 d. Which of the following telescopes might observe one of the photons?
  - JWST
  - WISE
  - EUVE
  - Chandra

2 e. Estimate very roughly the temperature of the material in the shocked region.

a) $KE = \frac{1}{2} m_{He} \cdot V^2 = \frac{1}{2}(4 * 1.67 \times 10^{-27}\ kg)(600 \times 10^3\ m/s)^2$

$= 1.20 \times 10^{-15}\ J$

b) for strong shock, atoms lose 3/4 of their speed, keep 1/4 of speed

→ keep 1/16 of KE

→ lose 15/16 of KE

$KE_{lost} = \frac{15}{16} KE = 1.13 \times 10^{-15}\ J * \frac{1\ eV}{1.609 \times 10^{-19}\ J} \cong 7000\ eV$

$\approx 7\ keV$

c) if all the lost energy turned into one photon

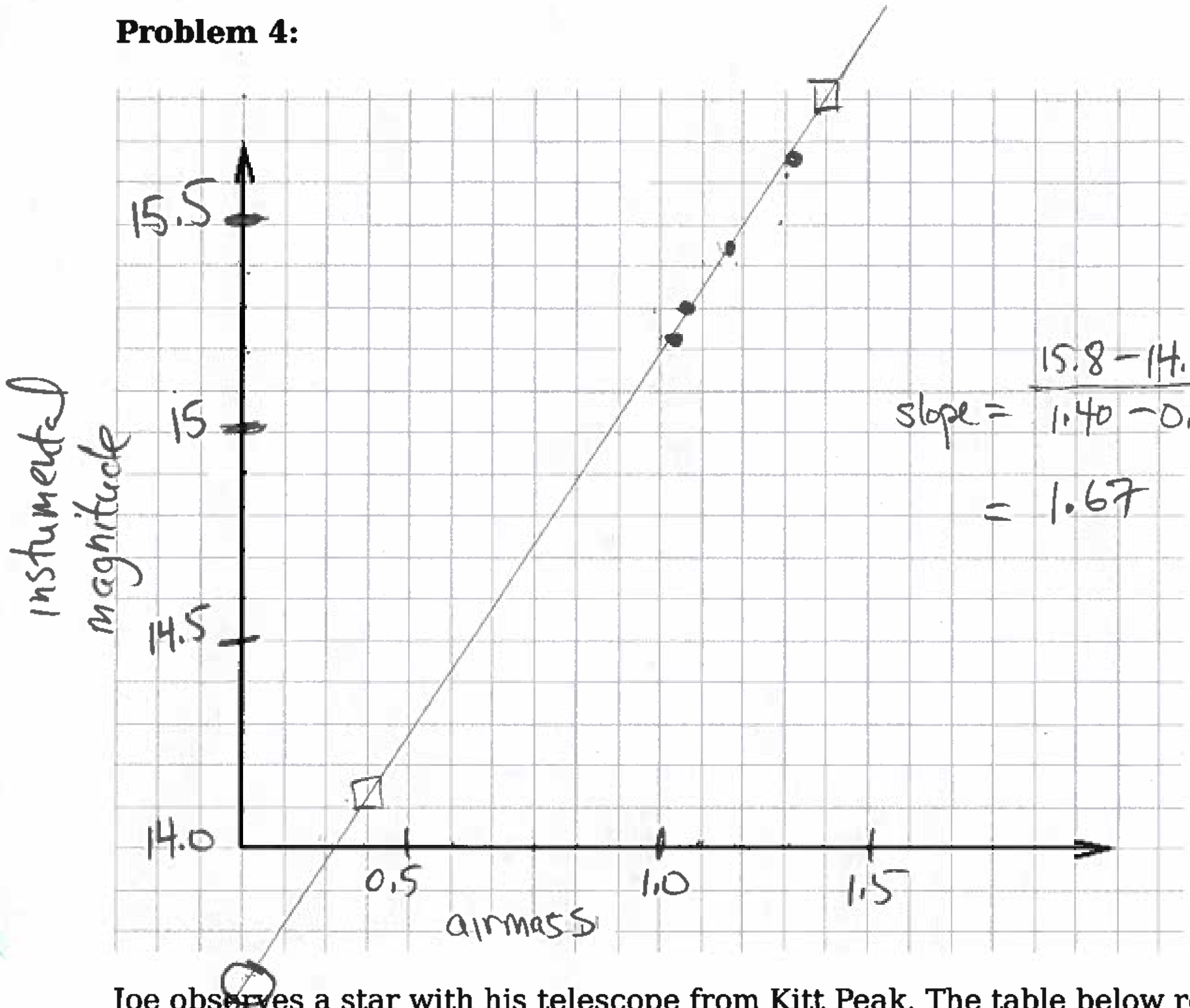
$\lambda = \frac{hc}{E} = \frac{1240\ eV \cdot nm}{7000\ eV} = 0.177\ nm$

d) Chandra

e) $KE_f = \frac{1}{16} KE_i = \frac{3}{2} kT \rightarrow T = \left(\frac{2}{3k}\right)\left(\frac{1}{16} KE_i\right)$

$T = \left(\frac{2}{3 * 1.38 \times 10^{-23}}\right)\left(\frac{1}{16} * 1.20 \times 10^{-15}\ J\right) = \approx 5.5 \times 10^7\ K$

Problem 4:



$$\text{slope} = \frac{15.8 - 14.13 \text{ mag}}{1.40 - 0.4 \text{ airmass}} = 1.67$$

Joe observes a star with his telescope from Kitt Peak. The table below records measurements of that star at four different times during the night, when it was at four altitudes above the horizon.

2 a. Fill in the column of the table labelled "Airmass"
5 b. Plot magnitude versus airmass on the graph above. Label axes appropriately.
2 c. What is the extinction coefficent for Joe's observations?
1 d. If Joe could move his telescope up above the Earth's atmosphere, what instrumental magnitude would he measure?

$$\text{airmass} = \frac{1}{\cos(90-a}$$

time (UT)	altitude above horizon (degrees)	airmass	instrumental magnitude
05:23	50	1.31	15.66
05:49	60	1.16	15.43
06:09	70	1.06	15.30
06:28	80	1.02	15.22

c) slope = 1.67 mag/airmass = ext coeff K

d) above atmosphere, airmass = zero (not 1)

on graph above, y-intercept at mag ≈ 13.7