ASTP613 Final Exam Practice Problem

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EuropaCube:

The year is 2325 and human civilization has developed space-faring technology that greatly surpasses the rudimentary capabilities of previous centuries. Multimessenger astronomy has likewise advanced into a new era, yet ultra-high redshift supernovae remain elusive to astronomers. The international collaboration JANE-SNOW[1] has begun a new mission to build a "next generation IceCube" water cherenkov neutrino observatory on the surface of the galilean moon Europa. The extremely high salinity of Europa's surface ice contains 15% more ions than terrestrial ice, making it more efficient at detecting neutrinos!

Part A: Derive an expression for the minimum surface area required to detect an ultra-high redshift $z = 12$ core collapse supernova with EuropaCube.

Part B: Can EuropaCube detect the supernova in part A and comfortably fit on the surface of Europa? If so, what percentage of the surface of Europa would be covered by boreholes?

In answering the parts above you may make the following assumptions and any other simplifying assumption necessary to give a solution.

(1) $N_\nu = 3.0 \times 10^{58}$ neutrinos with energy $E_\nu = 10\ MeV$ are released in a single CCSN neutrino burst.

(2) $d_{SN} = 130,540\ Mpc$ is the luminosity distance of an ultra-high redshift supernova at $z = 12$.

(3) $y = 2.5\ km$ is the maximum depth our best borehole drilling robots can reach into the surface ice of Europa.

(4) $R_E = 1,560\ km$ is the equatorial radius of Europa.

(5) $\eta = 1.15$ is the salty ions coefficient, a necessary prefactor for the increased efficiency of neutrino detections from EuropaCube.

(6) $\rho_{ice} = 917\ kg \cdot m^{-3}$ is the mass density of ice.

(7) $\chi_p = 10/18$ is the mass fraction of protons in liquid water.

(8) $N_{int} = 4$ is the number of neutrino interactions required in a small amount of time to produce a positive detection of a CCSN neutrino event – just one neutrino is not enough!

[1]**<u>JA</u>**XA, **<u>N</u>**ASA, and **<u>E</u>**SA **<u>S</u>**uper**<u>N</u>**ova **<u>O</u>**bservatories on other **<u>W</u>**orlds

Solution:

Part A:

The number of interactions N_{int} and the effective interaction area A are given by,

$$N_{int} = F_\nu \cdot A \tag{1}$$

$$A = N_{e^-} \cdot \sigma_{e^-,\nu} \tag{2}$$

Then the number of electrons N_{e^-} is equal to the number of protons multiplied by the salty ions coefficient η like,

$$N_{e^-} = \frac{M_{ice}}{m_p} \chi_p \cdot \eta \tag{3}$$

and the intensity of neutrinos from the burst is given by the familiar inverse square law.

$$F_\nu = \frac{N_\nu}{4\pi\, d_{SN}^2} \tag{4}$$

We can write an expression for the mass of the ice in the detector as,

$$M_{ice} = \rho_{ice} \cdot V_{ice} = \rho_{ice}\, y\, a_{surf} \tag{5}$$

where the surface area a_{surf} is the desired quantity.

Now we can combine equations to write new expressions for the quantities given in equations (1) and (2).

$$A = \frac{\rho_{ice}\, y\, a_{surf}}{m_p} \chi_p\, \eta\, \sigma_{e^-,\nu} \tag{5}$$

$$N_{int} = \frac{\rho_{ice}\, y\, a_{surf}\, \chi_p\, \eta\, \sigma_{e^-,\nu}}{4\pi\, d_{SN}^2\, m_p} = 4 \tag{6}$$

Rearranging for the surface area gives the final expression,

$$\boxed{a_{surf} = \frac{16\pi\, d_{SN}^2\, m_p}{\rho_{ice}\, y\, \chi_p\, \eta\, \sigma_{e^-,\nu}\, N_\nu}}$$

QED

Solution: (cont.)

Part B:

Utilizing the result from part A and the given numerical quantities (with known constants from class lectures),

$$a_{surf} = \frac{(16\pi)(130540 \times 10^6\ pc\ \cdot\ 3.086 \times 10^{16}\ m \cdot pc^{-1})^2 (1.67 \times 10^{-27}\ kg)}{(917\ kg \cdot m^{-3})(2.5 \times 10^3\ m)(10/18)(1.15)(9.0 \times 10^{-48}\ m^2)(3.0 \times 10^{58})} \tag{1}$$

$$a_{surf} = 3.44 \times 10^{12}\ m^2 \ \text{ or } \ \approx\ (1.854 \times 10^6\ m)^2 \tag{2}$$

Corresponding to a square array of boreholes with side length $\approx 1,854\ km$, <u>the EuropaCube detector could absolutely fit on the surface of Europa!</u>

Finally, we can calculate a surface area percentage as follows,

$$a_E = 4\pi\, R_E^2 = (4\pi)(1560 \times 10^3)^2 \tag{3}$$

$$a_E = 9.73 \times 10^{13}\ m^2 \tag{4}$$

$$\implies p = 100 \times \frac{a_{surf}}{a_E} = (100)\frac{(3.44 \times 10^{12}\ m^2)}{(9.73 \times 10^{13}\ m^2)} \tag{5}$$

$$\boxed{p = 3.54\%}$$

QED