

A pair of $2M_{\odot}$ neutron stars merge at a speed of $0.15c$ and the resulting gravitational wave has a distance modulus of 36. Will LIGO detect this event? LIGO has a strain sensitivity of $h \sim 10^{-22}$ and an arm length of 4km.

$$m - M = 5 \log_{10}(d) - 5 = 36$$

$$d = 1.58 \times 10^8 \text{ pc} \sim 158 \text{ Mpc} = 4.88 \times 10^{24} \text{ m} \quad (1 \text{ pc} = 3.086 \times 10^{16} \text{ m})$$

$$h \sim \frac{GM}{c^2} \frac{1}{D} \left(\frac{v}{c} \right)^2 = \frac{(6.674 \times 10^{-11})(2(2 \times 10^{30}))}{(3 \times 10^8)^2} \cdot \frac{1}{(4.88 \times 10^{24})} \left(\frac{0.15(3 \times 10^8)}{3 \times 10^8} \right)^2$$

$$\rightarrow h = 1.4 \times 10^{-23} \rightarrow h_{\text{LIGO}} \sim 10^{-22} > 1.4 \times 10^{-23}$$

so LIGO wouldn't detect this BNS merger

To detect this signal, for how long would a photon need to travel before interfering with another?

$$\Delta x = hL, L = ct$$

$$h_{\text{LIGO}} L = h(ct)$$

$$\rightarrow t = \frac{h_{\text{LIGO}} L}{hc} = \frac{(10^{-22})(4000)}{(1.4 \times 10^{-23})(3 \times 10^8)} = 9.5 \times 10^{-5} \text{ s} \sim 0.1 \text{ ms}$$