

practice problem:

a) pixl size $p = L \tan \alpha$, $p = 5.4 \mu\text{m}$, $L = 24 \text{mm}$, $\arctan(p/L) = \alpha \therefore \alpha = 0.75''$

b) $\theta = 1.22 \lambda / D$, $D = 8 \text{mm}$, $\lambda = 550 \text{nm}$ $\therefore \theta_{\text{optical}} \geq 1.22 \frac{550 \text{nm}}{8 \text{mm}} = 17.3''$. The resolution of the eye is diffraction limited.

c) $R_0 = 7 \cdot 10^{10} \text{cm} \rightarrow d_0 = 1.4 \cdot 10^{11} \text{cm}$, we know $\text{parsec} \equiv \text{AU} / \text{arcsec}$ and $1.4 \cdot 10^{11} \text{cm} = 9.3 \cdot 10^{-3} \text{AU}$

$$9.3 \cdot 10^{-3} / 17.3'' = 5.4 \cdot 10^{-4} \text{pc}$$

there are no stars (other than the sun) which are this close to Earth

d) The diffraction limit of the eye means the image of the disk of the star is spread over multiple rod cells.*

* of course, atmospheric blurring also contributes to this effect of spreading light over multiple cells, but I believe (I hope) that this effect is less significant than the diffraction limit

e) $17.3''$ is the diffraction limited resolution, $0.75''$ is angular resolution of rod cell.

$17.3'' / 0.75'' =$ diffraction-limited circle is 23 rods across \therefore has an area equal to ~ 415 rod cells.

min 6 γ/s per rod cell $\rightarrow 2490 \gamma/\text{s}$ must enter the eye for a star to be visible

aperture of eye is 8mm diameter \rightarrow area of "light-bucket" is 0.5cm^2

$$\frac{2490}{0.5} = 4980 \gamma/\text{cm}^2 \text{ is the minimum photon flux incident upon the eye}$$

$$4980 \gamma/\text{cm}^2 \rightarrow 4980 \gamma/\text{s} = L_2$$

multiply by cm^2 (*)

$$\frac{L_1}{L_2} = 10^{0.4(m_2 - m_1)}$$

$$10^6 \gamma/\text{cm}^2 \rightarrow 10^6 \gamma/\text{s} = L_1 \rightarrow m_1 = 0$$

$$\frac{L_1}{L_2} = 10^{0.4(m_2 - m_1)} \rightarrow \frac{10^6}{4980} = 10^{0.4m_2} \rightarrow \log_{10}\left(\frac{10^6}{4980}\right) \left(\frac{1}{0.4}\right) = m_2 = \text{largest apparent magnitude a preceptible star can have} = 5.75$$

* this is a sneaky trick that works because we only care about the ratio of luminosities,

not the true values. $L = F \cdot \text{area} \rightarrow L \propto F \rightarrow \frac{L_1}{L_2} = \frac{F_1}{F_2}$