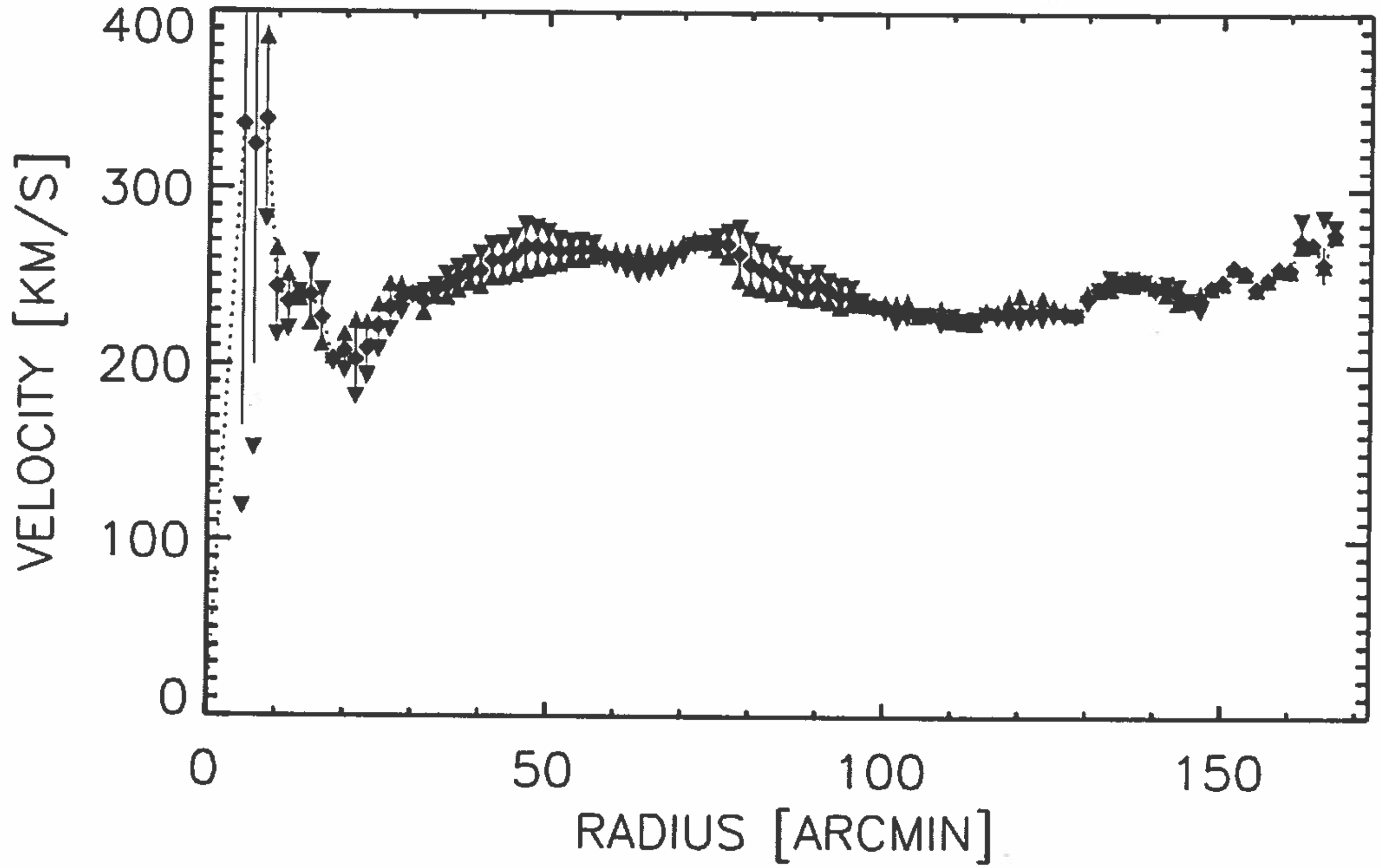
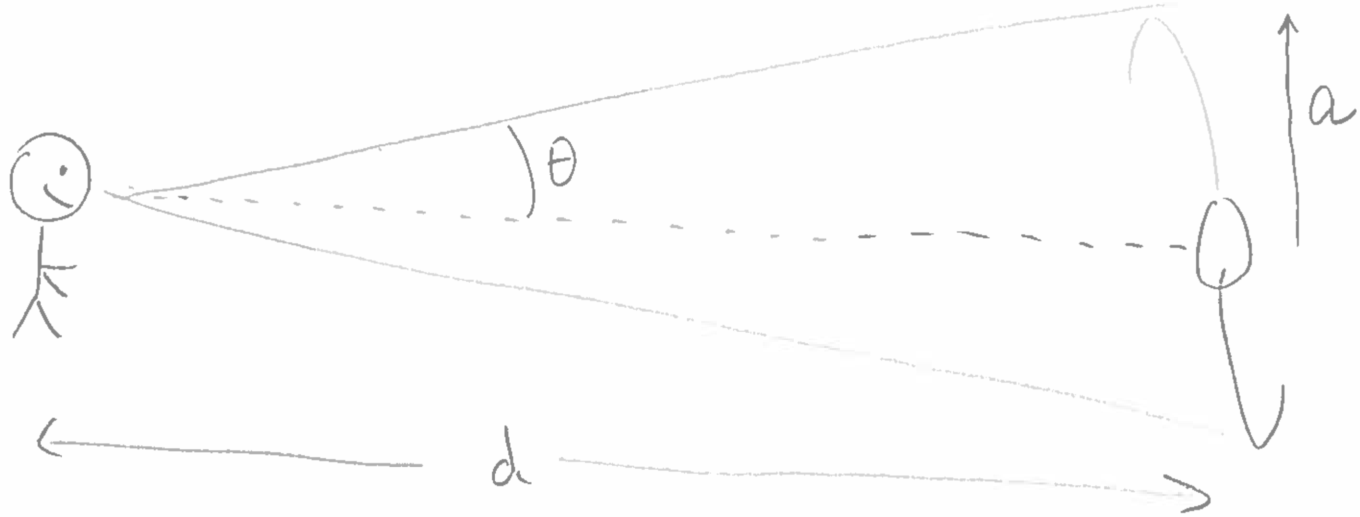


FIG. 10.— HI rotation curve of Messier 31. Filled diamonds are for both halves of the disc fitted simultaneously while blue downward/red upward triangles are for the approaching/receding sides fitted separately (respectively).



Given $d = 800$ kpc. From rotation curve, the outermost point shows

$$V = 280 \ \frac{km}{s} \quad \text{at radius } \theta = 160 \text{ arcmin}$$
$$= 9600''$$

The linear distance from center of M31 to this outermost point is

$$a = d\theta = (800 \times 10^3 \ pc)(9600'')$$
$$= 7.68 \times 10^9 \ AU$$

How long does it take gas to complete one orbit at this distance? Compare to the period of Earth around Sun:

$$\frac{P(\text{gas in M31})}{P(\text{Earth around Sun})} = \frac{a(\text{gas})}{a(\text{Earth})} \Big/ \frac{V(\text{gas})}{V(\text{Earth})}$$

$$= 7.68 \times 10^9 \Big/ \frac{280 \ km/s}{29 \ km/s}$$

$$\frac{P\ (\text{gas in M31})}{P\ (\text{Earth around Sun})} = \frac{7.68 \times 10^9}{9.65} = 7.9 \times 10^8$$

$$\rightarrow P\ (\text{gas in M31}) = 7.9 \times 10^8 \text{ yr}$$

Now use Kepler's Third Law to estimate mass of M31 internal to this outermost gas. This assumes material is distributed in a spherical form, but is at least an easy calculation.

$$P^2 = \frac{1}{M} a^3$$

$$\rightarrow M = \frac{a^3\ (\text{AU})}{P^2\ (\text{yr})} = \frac{(7.68 \times 10^9 \text{ AU})^3}{(7.9 \times 10^8 \text{ yr})^2}$$

solar masses

$$\boxed{M = 7 \times 10^{11}\ M_\odot}$$