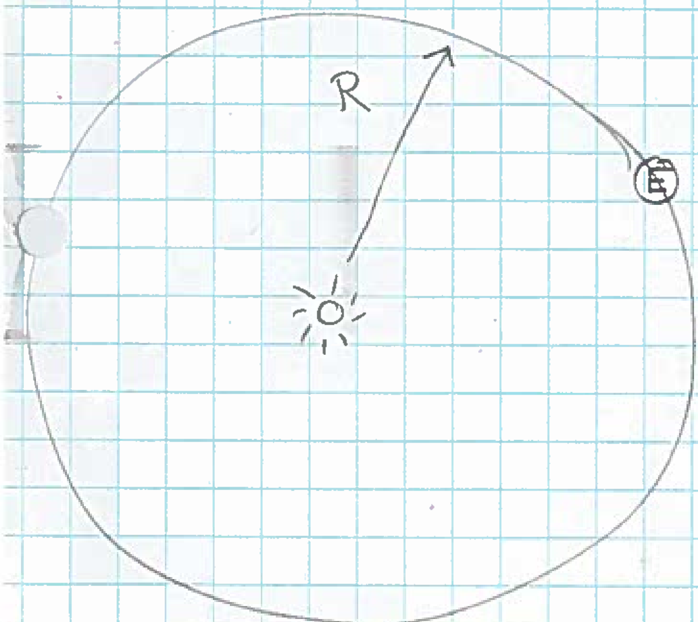
The Sun spontaneously turns into a (big, fluffy) neutron star!

Every proton merges with a nearby electron.

How many interactions with neutrinos are recorded in the SAGE detector?

The number of proton (and electrons) in the Sun is roughly

$$N_p = \frac{M_\odot}{m_p} = \frac{2 \times 10^{30}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}} \cong 1.2 \times 10^{57}$$



At the distance of the Earth, the (time-integrated) flux of neutrinos must be

$$F_\nu = \frac{N_p}{4\pi R^2} \quad \frac{\text{neutrinos}}{m^2}$$

$$= \frac{1.2 \times 10^{57}\ \nu}{4\pi (1.5 \times 10^{11}\ m)^2}$$

$$= 4.2 \times 10^{33}\ \nu / m^2$$

[We assume each $p^+, e^- \rightarrow n, \nu$]

Now, we need to compute the total cross-section area of the SAGE detector. Using values from the lecture notes,

$$M \cong 50 \times 10^3 \text{ kg of } Ga^{71}$$

So

$$N_n \cong \frac{M}{m_{p^+}} \left(\frac{40\, n}{31\, p^+ + 40\, n} \right)$$

$$\cong 1.7 \times 10^{31} \text{ neutrons}$$

Using the cross section for neutrino-neutron collision of

$$\sigma = 10^{-50} \; m^2 \qquad \text{from the lecture}$$

the total area is

$$\sigma_{tot} = \sigma N_n = 10^{-50} m^2 \left(1.7 \times 10^{31} \text{ neutrons}\right)$$

$$= 1.7 \times 10^{-19} \; m^2$$

So, the number of interactions inside SAGE would be

$$N_{inter} = \sigma_{tot} F_\nu$$

$$= \left(1.7 \times 10^{-19} m^2\right)\left(4.2 \times 10^{33} \; \nu / m^2\right)$$

$$\boxed{N_{inter} \cong 7 \times 10^{14}}$$

Can we estimate the number of interactions between neutrinos and neutrons inside a typical human body?

$$N_n \cong \frac{m_{human}}{m_p} \cdot \frac{1}{2}$$

assuming most elements in human body are half proton / half neutron

C_{12}, O_{16}, for example

So, roughly

$$N_n \cong \frac{60\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} \cdot \frac{1}{2}$$

$$\cong 1.8\times10^{28}\ \text{neutrons}\ldots$$

Number of interactions will be

$$N_{inter} \cong \left(1.8\times10^{28}\ \text{neutrons}\right)\left(10^{-50}\ \frac{m^2}{\text{neutron}}\right)\left(4.2\times10^{33}\ \frac{\nu}{m^2}\right)$$

$$= \left(1.8\times10^{-22}\ m^2\right)\left(4.2\times10^{33}\ \nu/m^2\right)$$

$$= 7.5\times10^{11}$$

Assuming further that each neutrino has a typical energy of

$$E_\nu \sim 0.3\ \text{MeV} \cong 5\times10^{-14}\ J$$

the total energy transferred to the human body is

$$E_{tot} \sim \left(5\times10^{-14}\ \frac{J}{\text{inter}}\right)\left(7.5\times10^{11}\ \text{inter}\right)$$

$$\sim 3.5\times10^{-2}\ J$$

The heat capacity of water is

$$c = 4186\ \frac{J}{kg\cdot{}^\circ C}$$

required to heat 1 kg by 1 °C

So the rise in body temperature would be ... small.

$$\Delta T \sim \frac{E_{tot}}{c\cdot m_{human}} = \frac{0.035\ J}{4186\ \frac{J}{kg\cdot{}^\circ C}\cdot 60\ kg} \cong 10^{-7}\ {}^\circ C$$