#4

The Crab Pulsar has a period of $P = 33$ msec $= 0.033$ sec. How easy is it to detect the pulses of this object in γ-rays, and in the optical regime? Let's consider the statistics of photons...

a) In optical, at $\lambda = 7000$ Å, the flux of Crab Pulsar is

$$f_{opt} = 3 \times 10^{-12} \ \frac{\text{erg}}{\text{sq.cm} \cdot \text{s}}$$

So if we use a telescope of aperture $R = 50$ cm, and pretend that all the photons we detect are exactly $\lambda = 7000$ Å

$$\text{collecting area } A = \pi R^2 = 7854 \text{ cm}^2$$
$$\text{energy per photon } E = hc/\lambda = 2.84 \times 10^{-12} \text{ ergs}$$

We collect this many photons per second

$$N(\text{opt}) = \frac{f \cdot A}{E} = \left(\frac{3 \times 10^{-12} \ \frac{\text{erg}}{\text{cm}^2 \cdot \text{s}}}{2.84 \times 10^{-12} \text{ ergs}}\right)\left(7854 \text{ cm}^2\right)$$

$$\approx \boxed{8300 \ \frac{\text{photon}}{\text{s}}}$$

b) Each second contains $\frac{1}{P} = 30.3$ flashes, so the number of photons per flash is

$$N(\text{per flash}) \approx \frac{8300}{30.3} \approx \boxed{270 \ \frac{\text{photons}}{\text{flash}}}$$

#4 part 2

c) Now consider the γ-ray emission. For the sake of argument, pretend that the pulsar emits exactly the same flux in γ-rays of energy

$$E_\gamma = 10\,\text{MeV} = 10\times10^6\,\text{eV}$$

$$= 10^7\,\text{eV} \cdot 1.609\times10^{-12}\,\frac{\text{erg}}{\text{eV}}$$

$$= 1.6\times10^{-5}\ \text{ergs per } \gamma\text{-ray}$$

$$\#\,\gamma\text{-rays per second } N_\gamma = \frac{3\times10^{-12}\,\frac{\text{erg}}{\text{cm}^2\cdot\text{s}} \times (7854\ \text{cm}^2)}{1.6\times10^{-5}\ \text{erg}/\gamma\text{-ray}}$$

$$\boxed{N_\gamma = 1.47\times10^{-3}\ \gamma\text{-ray/second}}$$

d) If there are 30.3 flashes per second, then each flash must contain

$$N \text{ per flash in } \gamma\text{-ray} = \frac{1.47\times10^{-3}}{30.3}$$

$$\simeq \boxed{4.9\times10^{-5}\ \frac{\gamma\text{-ray}}{\text{flash}}}$$

It will be very hard to detect a periodic signal in γ-rays — but easy in optical photons.