The Solar Spectrum 2008: Gaussianity of Absorption Lines
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Fitting models for Spectrum lines - Graduate Seminar I

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Abstract. The aim of this paper is to fit a model for an absorption line. We work with a typical absorption line in the solar spectrum, and propose several parametric models (Gaussian, Lorentzian, Voigt profile) to fit the data. The results show that even if we can access certain success with these models, the field is much more complicate and can not be reduced to a simply fitting exercise.

1. Introduction

The spectrum of the Sun, like that of most stars, includes many absorption lines. Understand how this lines and the shape of their profiles is a great challenge to understand their composition and chemical properties. Therefore there is a important effort in order to get more accurate models in order to explain and describe such spectrum line profiles. We can find in the literature works mainly related to understand the chemical composition of the stars (2), atmosphere properties (5) or even more technical ones related to specific package in order to fit special profiles.

Among the most common effects for analyze the profile lines are: atomic effects, radiation broadening, radiation damping, Doppler broadening, collisional broadening and other combined effects (1).

In this paper we analyze the data coming from a typical Solar line, fitting it with a Gaussian model. This paper is organized as follows: in section 2. we briefly describe the model used, in section 3. we show the results for the numerical fitting and in section 4. we present the conclusions and discussion.

2. Gaussian Model for line shape

One of the simplest models to try to describe the shape of the spectral lines is a Gaussian distribution. In such model the advantage is that mathematical details are mostly simple and it only deals with four parameters,

$$I(\lambda) = C - De^{\frac{-(\lambda-\lambda_0)^2}{2\sigma^2}}. \tag{1}$$

Eq. (1) shows the Gaussian model for the intensity of an absorption line as a function of the wave length (λ), where C is the level of the continuum, D is the depth of the line, λ_0 is the central wavelength and σ^2 is related with the Full-Width at Half-Maximum of the line (FWHM, which is equal to 2.35σ).

One way to describe the degree to which some model fits a dataset is to calculate the "χ-squared" statistic, defined in Eq. (2),

$$\chi^2 \equiv \sum_i \frac{(x_i - y_i)^2}{(\Delta_i)^2} \tag{2}$$

where x_i are the values coming from the model, y_i the experimental data, and Δ_i the uncertainty for each measure i. Actually, in order to get a good estimator of the matching, the *reduced χ-squared* statistic must be computed, which reads as the χ^2 over the number of degrees of freedom in the problem.

3. Fitting the Spectral line with the Gaussian model

Using data coming from the "Solar Survey Archive" (BASS2000), we access to measures for a typical spectral line for the Sun. As we can see in Fig. 1, the data distribution can be thought that obeys a Gaussian distribution, mainly in the central region. Therefore, taking this into account we try a Gaussian fit for the "inner" region (i.e. where λ runs from 6546.12Å to 6546.38Å).

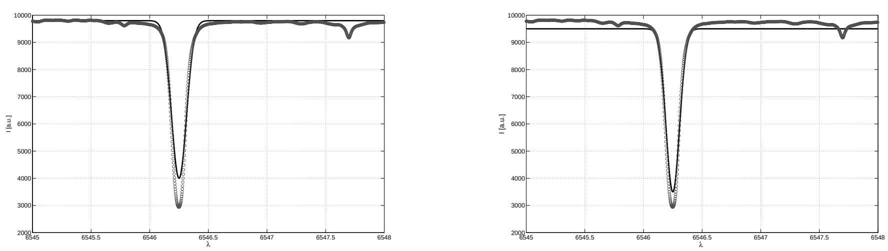

Figure 1. Empirical profile (grey circles) and Gaussian fit (black solid line), guessing parameters by inspection on the left panel and with the "best fit" on the right panel; for a typical absorption line in the solar spectrum. Wavelength (λ) units are amstrongs (Å), and intensity (I) units are arbitrary.

The experimental data and the fits (a guess fit and the best-one) for the Gaussian model, are shown in Fig. 1.

Table 1. Parameters value for the best-fit Gaussian model.

fit	C (a.u.)	D (a.u.)	λ_0 (Å)	σ [FWHM = 2.35σ]	reduced-χ^2 [χ^2/dof]
#1	9800	5800	6546.25	0.0638	6.55×10^7
#2	9500	6000	6546.2	0.0574	6.83×10^5

In Table 1, we show the four parameters for the Gaussian distribution used to fit this spectral profile. A very simple iterative routine was developed in order to look for the best parameters values such that minimize the reduced-χ^2 value, although it does not improve too much the initial fitness. In spite of that, the best-fit doesn't enhance the situation regarding the χ^2 indicator as well as the graphic profile neither. As it can be observed in Fig. 1, the differences between the panels and the fitness models are mostly in the 'base-line' due to the guest value for the parameter C, it also makes possible to get closest values for the minimum of the spectrum.

4. Discussion

According to the parameters values, in particular to the reduced-χ^2 indicator, the Gaussian distribution is not a very good model for this spectral profile. In addition to this, many authors have developed more sophisticated algorithms and models in order to explain the complicates line profiles observed in the atmosphere spectrum.

One of the most popular and well-known models is the *Voigt profile*. A Voigt profile function emerges in several physical investigations (e.g. atmospheric radiative transfer, astrophysical spectroscopy, plasma waves and acoustics) and it turns out to be the convolution of the Gaussian and the Lorentzian densities (4). Also this profile is motivated for more phenomenological and fundamental explanations related to the nature of the absorption and spectral lines.

In spite of that, many astronomers still use more simple models as the Gaussian one, and they also try to add more realistic facts to the model. For instance, taking into account the empirical data presented in Fig. 1, there are some small peaks far from the 'inner region' where we can assume a Gaussian behavior. Some authors had proposed to explain these peaks with the randomly addition of noise to the model (6).

Summarizing, we must argue that the Gaussian model is more simple and easy to compute although the results are not as good and precise as achieve with more complicate models.

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